# CHAPTER 2

# Risk & Return

## Chapter Outline

THIS CHAPTER CONTAINS THE FOLLOWING CONCEPTS;

- *Definitions*
- *Relation between risk and return*
- *Difference between risk and uncertainty*
- *Sources of risk*
- *Risk preference*
- *Measurement of risk & return in individual and in portfolio context*
- *Measurement of total risk*
- *Efficient portfolio*
- *Choosing the optimal portfolio*
- *Capital asset pricing model (CAPM)*
- *SML & CML*
- *APT*
- *Mathematical Problems & Solutions (NU BBA 6th Semester Final Examinations)*
- *Solved Problems*

## Chapter Objectives

AFTER READING THIS CHAPTER:

- *You will be able to define and illustrate different concepts of risk & return.*
- *You will be capable of describing the popular sources of risk & risk preference behaviors of investors.*
- *You will be competent to compare between risk & uncertainty.*
- *You will be proficient in taking proper steps in measuring stand alone risk as well as risk & return in a portfolio context.*
- *You will be expert in describing the relationship among and between risk and return, forming efficient portfolio, and choosing the optimal portfolio.*
- *You will be able to describe the Capital Asset Pricing Model (CAPM), Arbitrage Pricing Theory (APT) and the significance of risk in the field of financial environment.*

**"Take calculated risks. That is quite different from being rash."**

**GENERAL GEORGE S. PATTON**

* Risk is the variability of actual return from the expected return.

## 2.1 INTRODUCTION

Finance literature speaks that there is a positive relationship between risk and return. In a nutshell, higher risk investments must offer investors higher potential returns because investors, as a group, are risk averse. Risk aversion has two general implications. The first is that people will pay to avoid risk – something we all do when we buy insurance. We transfer risk from ourselves to the insurance company, in exchange for a fee (the insurance premium). Risk aversion also implies that people will accept risk only if they are offered some sort of incentive. Higher potential returns are one such stimulus.

## 2.2 RETURN

Return is defined as any outcome of an investment. In another way return is the total gain or loss experienced on an investment over a given period of time. It is commonly measured as cash distributions during the period plus the change in value, expressed as a percentage of the beginning-of-period investment value.

Investments offer two potential sources of returns which are income and price changes. Income is the periodic cash flow paid the investor. Some investments pay no income, and others pay a relatively high amount. Some investments pay a fixed amount each year; other investment incomes can vary quite a bit from year to year. Periodic income received from stock investments is called dividends. The periodic income received from bonds and money market instruments is referred to as interest. The other source of investment returns is price changes. We refer to an increase in the price of an investment as a capital gain; a decrease in the price of an investment is referred to as capital loss.

### 2.2.1 Components of Return

Investments offer two potential sources of returns, income and price changes to be more specific yield and capital gains or loss.

***I. Yield or Income*** is the periodic cash flow paid to the investor. Some investments pay no income, and others pay a relatively high amount. Some investments pay a fixed amount each year; other investment incomes can vary quite a bit from year to year. Periodic income received from stock investments is called dividends. The periodic income received from bonds and money market instruments is referred to as interest.

2. ***Capital Gain or Loss:*** The other source of investment returns is price changes or capital gain or loss of the investment. We refer to an increase in the price of an investment as a capital gain; a decrease in the price of an investment is referred to as capital loss.

***Return of an Investment:*** The rate of return of an investment can be found by applying the following formula; for the simplicity we are considering the holding period of investment is one year.

$$R_t = \frac{CF_t + P_t - P_{t-1}}{P_{t-1}}$$

Where,

| | | |
|---|---|---|
| $R_t$ | = | actual, expected, or required rate of return during period *t* |
| $CF_t$ | = | cash flow received from the asset investment in the time period *t* – 1 to *t* |
| $P_t$ | = | price (value) of asset at time *t* |
| $P_{t-1}$ | = | price (value) of asset at time *t* – 1 |

***Example:*** Suppose Mr. Rhyme has purchased a share of Beximco Pharmaceuticals Ltd. in the January 2012 at Tk. 100 and at the end of the year the company paid a dividend of Tk. 20 for the year 2012, if the market price of the share in the December 2012 is 120. What would be the rate of return on investment for the year?

$$R_{2012} = \frac{CF_{2012} + P_{December} - P_{January}}{P_{January}}$$

$$= \frac{20 + 120 - 100}{100} = .40 \text{ or } 40\%$$

$CF_t$ = Tk. 20

$P_t$ = Tk. 120

$P_{t-1}$ = Tk. 100

### 2.2.2 Ex-Ante Vs Ex-Post Returns

Between the beginning of 1990 and the end of 1999, stock returns have averaged close to 17 percent per year. What kind of return is this? It's a historical return, also known as an ex-post return. It's what someone would have earned had they invested in stocks during the 1990s. Let's say, an analyst forecasts that stocks will return an average of 15 percent over the next ten years. This return is an ex-ante return. It's the return we expect to earn over some future period of time.

### 2.2.3 Expected Return

There are several measures of central tendency, but finance emphasizes the importance of a particular measure, called the expected value of a random variable, which is the probability-weighted average of the possible outcomes or the mean of the probability distribution of random variable. Expected return can be defined as the probability-weighted average of the possible returns or the mean of the probability distribution of returns. Mathematically, we can calculate two types of expected return; based on probabilistic data and another one is based on historical data.

***Expected return based on probabilistic data;***

$$\text{Expected Return} / E(R_i) / \overline{R} = \sum_{i=1}^{n} R_i \times P_i$$

*Where,*

| | | |
|---|---|---|
| $R_i$ | = | return for the ith outcome |
| $P_i$ | = | probability of occurrence of the ith outcome |
| $n$ | = | number of outcomes considered |

***Expected return based on historical data;***

$$\text{Expected Return} / E(R_i) / \overline{R} = \frac{\sum_{i=1}^{n} R_i}{n}$$

*Where,*

| | | |
|---|---|---|
| $R_i$ | = | return for the ith outcome |
| $n$ | = | number of outcomes considered |

## 2.3 RISK

Risk is the variability of actual return from expected return.

Risk is the uncertainty of an investment's actual return in the future. Risk is a concept which relates to human expectations. It denotes a potential negative impact to an asset or some characteristic of value that may arise from some present process or from some future event. In everyday usage, "risk" is often used synonymously with "probability" of a loss or threat. Risk, in and of itself, is neither good nor bad; it merely exists. Risk is also often a double-edged sword, having an upside as well as downside; it is like the sugar and salt of life.

*"Risk is defined as the chance that the actual outcome of an investment will differ from the expected outcome."* – **C.P. Jones**

*"Risk is the chance that some unfavorable events will occur."* – **J.F. Weston & E.F. Brigham**

*"Risk is defined as the variability of returns associated with a given asset."* – **L.J. Gitman**

*Finally we can say that, There is no single definition of risk. Economists, behavioral scientists, risk theorists, statisticians, and actuaries each have their own concept of risk. However, risk traditionally has been defined in terms of uncertainty. Based on this concept, risk is defined here as uncertainty concerning the occurrence of a loss.*

### 2.3.1 Risk Vs Uncertainty

| Points of Distinction | Risk | Uncertainty |
|---|---|---|
| **Definition** | Risk is defined as the variability of returns associated with a given asset. | Uncertainty refers to a situation where the outcome is not certain or unknown. |
| **Probability Distribution** | Risk refers to a set of unique outcomes for a given event which can be assigned probabilities. | Probabilities cannot be assigned in case of uncertainty. |
| **Historical Data** | Risk occurs when the decision maker has some historical data on the basis of which he can calculate the probability of occurrence of any future outcomes. | In case of uncertainty there is no historical data available and we cannot measure the uncertainty. |
| **Management** | Risk can be avoided through different risk management techniques. | Uncertainty cannot be avoided through management. |
| **Relationship with return** | The relationship between risk and return is positive and linear. | There is no exact relationship can be expressed between uncertainty and return. |
| **Measurement** | Risk can be measured by using mathematical or statistical tools. | Uncertainty can not be measured. |

### 2.3.2 Sources of Risk

As we know that risk is related with human expectations and has been defined by different scholars in different view point. The potential sources of risk can be pointed out under the following heads.

#### 1. Firm Specific Risks

**Business risk:** The chance that the firm will be unable to cover its operating costs. The level of business risk is driven by the firm's revenue stability and the structure of its operating costs (fixed vs. variable).

**Financial risk:** The chance that the firm will be unable to cover its financial obligations. The level of financial risk is driven by the predictability of the firm's operating cash flows and its fixed cost financial obligations.

#### 2. Shareholder Specific Risks

**Interest rate risk:** The chance that changes in interest rates will adversely affect the value of an investment. Most investments lose value when the interest rate rises and increase in value when it falls.

**Liquidity risk:** The chance that an investment cannot be easily liquidated at a reasonable price. Liquidity is significantly affected by the size and depth of the market in which an investment is customarily traded.

**Market risk:** The chance that the value of an investment will decline because of market factors that are independent of the investment (such as economic, political, and social events). In general, the more a given investment's value responds to the market, the greater its risk; and the less it responds, the smaller its risk.

#### 3. Firm & Stockholder Risks

**Event risk:** The chance that a totally unexpected event will have a significant effect on the value of the firm or a specific investment. These infrequent events such as government-mandated withdrawal of a popular prescription drug, typically affect only a small group of firms or investments.

***Exchange rate risk:*** The exposure of future expected cash flows to fluctuations in the currency exchange rate. The greater the chance of undesirable exchange rate fluctuations, the greater the risk of the cash flows and therefore the lower the value of the firm or investment.

***Purchasing power risk:*** The chance that changing price levels caused by inflation or deflation in the economy will adversely affect the firm's or investment's cash flows and value. Typically, firms or investments with cash flows that move with general price levels have a low purchasing power risk, and those with cash flows that do not move with general price levels have high purchasing power risk.

***Tax risk:*** The chance, that unfavorable change in tax laws will occur. Firms and investments with values that are sensitive to tax law changes are more risky.

### 2.3.3 Risk Preferences

There are three basic risk preference behaviors – risk-indifferent, risk-averse, and risk-seeking. Feelings about risk differ among managers (and firms). Thus it is important to specify a generally acceptable level of risk. The three basic risk preference behaviors—risk-averse, risk-indifferent, and risk-seeking are depicted graphically in the following figure.

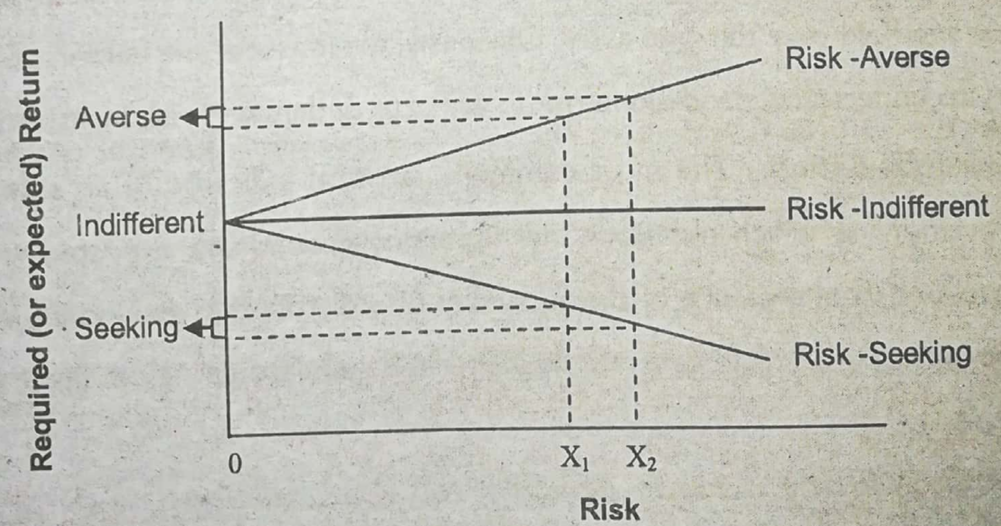



***Figure: Risk preference behavior***

***For the risk-indifferent manager***, the required return does not change as risk goes from $x_1$ to $x_2$. In essence, no change in return would be required for the increase in risk. Clearly, this attitude is illogical in almost any business context.

***For the risk-averse manager,*** the required return increases for an increase in risk. Because they shy away from risk, these managers require higher expected returns to compensate them for taking greater risk.

**For the risk-seeking manager,** the required return decreases for an increase in risk. Theoretically, because they enjoy risk, these managers are willing to give up some return to take more risk. However, such behavior would not be likely to benefit the firm.

***Most managers are risk-averse;*** for a given increase in risk, they require an increase in return. They generally tend to be conservative rather than aggressive when accepting risk for their firm. Accordingly, a risk-averse financial manager requiring higher returns for greater risk is assumed throughout this text.

## 2.4 STAND-ALONE RISK ANALYSIS OR RISK IN ISOLATION

An asset's risk can be analyzed in two ways: (1) on a stand-alone basis, where the asset is considered in isolation, and (2) on a portfolio basis, where the asset is held as one of a number of assets in a portfolio. Thus, an asset's stand-alone risk is the risk an investor would face if he or she held only this one asset. Obviously, most assets are held in portfolios, but it is necessary to understand stand-alone risk in order to understand risk in a portfolio context.

***The Standard Deviation:*** The most common statistical indicator of an asset's risk is the standard deviation, $\sigma_i$, which measures the dispersion around the expected value. Standard deviation is a statistical measure of the variability of a distribution around its mean. It is the square root of the variance. In general, the higher the standard deviation, the greater the risk is.

The expression for the standard deviation of returns for probabilistic return data, $\sigma_i$ is

$$\sigma_i = \sqrt{\sum_{i=1}^{n} \left(R_i - \overline{R}_i\right)^2 \times p_i}$$

When we calculate risk on the basis of historical data, $\sigma_i$ is calculated as,

$$\sigma_i = \sqrt{\frac{\sum_{i=1}^{n} \left(R_i - \overline{R}_i\right)^2}{n-1}}$$

Where, $R$ = mean return over the period

***The Variance:*** Variance is nothing but the sum of the square deviations of individual returns from the mean return. In the process of calculating standard deviation variance is found earlier and if we take the square root of variance we got the standard deviation. In general, the higher the variance the greater the risk is.

The expression for the variance of returns for probabilistic return data, is

$$\sigma^2{}_i = \sum_{i=1}^{n} \left(R_i - \overline{R}_i\right)^2 \times p_i$$

When we calculate risk on the basis of historical data, is calculated as,

$$\sigma^2{}_i = \frac{\sum_{i=1}^{n} \left(R_i - \overline{R}_i\right)^2}{n-1}$$

Where, $R$ = mean return over the period

***The Coefficient of Variation:*** The coefficient of variation is a measure of relative dispersion – a measure of risk per unit of expected return – that is useful in comparing the risks of assets with differing expected returns. This is a standardized method of measuring risk. It is calculated by dividing standard deviation with the expected return. The higher the coefficient of variation, the greater the relative risk. The expression for the coefficient of variation is;

$$CV = \frac{\sigma_i}{\overline{R}}$$

Where, CV= Coefficient of Variation

$\sigma_i$ = Standard deviation of stock *i*

$\overline{R}$ = The expected return of stock *i*

***Measuring Stand-Alone Risk: The Coefficient of Variation***

The standard deviation can sometimes be misleading in comparing risk, surrounding alternatives if they differ in size. Consider two investment opportunities, *A* and *B*, whose normal probability distribution of one-year returns have the following characteristics:

| | Investment A | Investment B |
|---|---|---|
| Expected return, $E(R_i)$ | 0.10 | 0.30 |
| Standard deviation, $\sigma$ | 0.06 | 0.08 |
| Coefficient of variation, *CV* | 0.60 | 0.27 |

Can we conclude that because the standard deviation of *B* is larger than that of *A*, it is the riskier investment? With standard deviation as our risk measure, we would have to. However, relative to the size of expected return, investment *A* has greater variation.

To adjust for the size, or scale, problem, the standard deviation can be divided by the expected return to compute the coefficient of variation (CV). The coefficient of variation is a measure of relative dispersion – a measure of risk per unit of expected return – that is useful in comparing the risks of assets with differing expected returns. The higher the coefficient of variation, the greater the relative risk. Using the *CV* as our risk measure, investment A with a return distribution *CV* of 0.60 is viewed as being more risky than investment B, whose *CV* equals only 0.27.

## SELF-TEST | REVIEW QUESTIONS | Concept Checkers

**Q - 2.1** What is return? What are the components of return and how can we calculate the return of an investment?

**Q - 2.2** What is risk? (NU BBA – 2009)

**Q - 2.3** Differentiate between risk and uncertainty.

**Q - 2.4** Briefly explain the different sources of risk. (NU BBA – 2013)

**Q - 2.5** How is the risk of an investment measured in standalone point of view or risk in isolation?

Or, How is the risk of an investment calculated? (NU BBA – 2009)

**Q - 2.6** Why is coefficient of variation considered to be a better measure of risk than standard deviation in comparing more than one asset?

**Q - 2.7** How will you explain the risk preference characteristics of investors?

## 2.5 RETURN AND RISK IN A PORTFOLIO CONTEXT

Investors rarely place their entire wealth into a single asset or investment. Rather, they construct a portfolio. Portfolio is a combination of two or more securities or assets. Investors form portfolio to minimize the risk of investment and/or to maximize the return. The objective of portfolio formation is achieved through forming an efficient set of assets which either will maximize the return at a given level of risk or will minimize the risk at a given level of return.

Theory of portfolio suggests that ***a portfolio should be a well diversified combination of assets from a range of assets from the market having negative correlations.*** If we can properly diversify our investment the unsystematic or company specific risks can be eliminated. So we can say that a well diversified portfolio assumes the systematic risk of investment only. In forming efficient portfolio, Markowitz efficient set and theory of optimal portfolio choice help investors and academicians to understand the zeal of portfolio formation.

### 2.5.1 Portfolio Return

The expected return of a portfolio is simply a weighted average of the expected returns of the securities constituting that portfolio. The weights are equal to the proportion of total funds invested in each security (the weights must sum to 100 percent). The general formula for the expected return of a portfolio, $E(R_p)$ is as follows:

$$\overline{R}_p = \sum_{j=1}^{n} w_{ij} \overline{R_{ij}}$$

Where,

$w_j$ = proportion or weight of funds invested in security i j

$\overline{R}_p$ = expected return on asset i j

$n$ = total number of different securities in the portfolio

Thus for a two asset portfolio, the expected return is;

$$\overline{R}_p = W_A \overline{R}_A + W_B \overline{R}_B$$

কর্মা–৩(ক)

Where,

$W_A$ = proportion or weight of funds invested in security A

$W_B$ = proportion or weight of funds invested in security B

$\overline{R}_A$ = Expected return on asset A

$\overline{R}_B$ = Expected return on asset B

$\overline{R}_p$ = expected return on the portfolio

### 2.5.2 Portfolio Risk

Unlike the situation with returns, the standard deviation of a portfolio, $\sigma_p$, is generally not a weighted average of the standard deviation of the individual securities in the portfolio, and each stock's contribution to the portfolio's standard deviation is not $x_i\sigma_i$. Indeed, it is theoretically possible to combine two stocks which are, individually, quite risky as measured by their standard deviations, and to form from these risky assets a portfolio which is completely riskless, with $\sigma_p$=0%. However, the standard deviation of returns of a portfolio can be calculated from its expected returns and associated probabilities using the following equation:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \left(R_{pi} - \overline{R}_p\right)^2 \times p_i}$$

***Covariance and the Correlation Coefficient***: Two key concepts in portfolio analysis are: (1) *covariance* and (2) the *correlation coefficient*. Covariance is a measure of the degree to which two variables move together relative to their individual mean values over time. The following equations can be used to find the covariance between stocks A and B:

$$COV_{AB} = \sum_{i=1}^{n} p_i \left(R_A - \overline{R}_A\right)\left(R_B - \overline{R}_B\right)$$

*Calculating covariance from historical return data;*

$$COV_{AB} = \frac{\sum_{i=1}^{n} \left(R_A - \overline{R}_A\right)\left(R_B - \overline{R}_B\right)}{n-1}.$$

If stocks A and B tend to move together, their covariance, $Cov_{A,B}$, will be positive. While if they tend to move counter to one another, $Cov_{A,B}$, will be negative. If their returns fluctuate randomly, $Cov_{A,B}$ could be either positive or negative, but in either event it will be close to zero.

The correlation coefficient standardizes the covariance by taking into consideration the variability of the two individual return series, as follows:

$$\rho_{AB} = \frac{COV_{AB}}{\sigma_A \sigma_B}$$

$\rho_{AB}$ = correlation coefficient between returns of two stocks A and B

$$COV_{AB} = \sigma_A \sigma_B \rho_{AB}$$

The sign of the correlation coefficient is the same as the sign of the covariance, so a positive (+) sign means that the variables move together, a negative (–) sign indicates that they move in opposite directions, and if $\varrho$ is close to zero, they move independently of one another. Moreover, the standardization process confines the correlation coefficient to values between -1.00 and +1.0. The correlation equation can be solved to find the covariance:

***Portfolio Standard Deviation***

The general formula for the standard deviation of a portfolio as derived by Harry Markowitz is as follows:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{i=1}^{n} w_i w_j Cov_{ij}}$$

This formula indicates that the standard deviation for a portfolio of assets is a function of the weighted average of the individual variances (where the weights are squared), plus the weighted covariances between all the assets in the portfolio. In a portfolio with a large number of securities, this formula reduces to the sum of the weighted covariances.

For a two asset portfolio:

$$\sigma_p = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 Cov_{ij}}$$

Or,

$$\sigma_p = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_1 \sigma_2 \rho_{ij}}$$

### 2.5.3 Portfolio Risk Vs Stand Alone Risk

| Points of Distinction | Stand alone risk | Portfolio risk |
|---|---|---|
| **Definition** | An asset's stand-alone risk is the risk an investor would face if he or she held only this one asset. | A portfolio risk of an investment is that where the asset is held as one of a number of assets in a portfolio. |
| **Risk Measures** | ▪ Individual standard deviation<br>▪ Individual Variance<br>▪ Individual Coefficient of variation | ▪ Portfolio standard deviation<br>▪ Portfolio Variance<br>▪ Portfolio Covariances |
| **Diversification** | Stand alone risk measures the undiversified risk of an individual asset. | A portfolio is constructed to diversify the risk arise from different sources. |
| **Formula** | $\sigma_i = \sqrt{\sum_{i=1}^{n} (R_i - \overline{R}_i)^2 \times p_i}$<br>$\sigma^2_i = \sqrt{\sum_{i=1}^{n} (R_i - \overline{R}_i)^2 \times p_i}$<br>$CV = \frac{\sigma_i}{\overline{R}}$ | $\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{ij}}$<br>$COV_{AB} = \sum_{i=1}^{n} p_i (R_A - \overline{R}_A)(R_B - \overline{R}_B)$<br>$COV_{AB} = \frac{\sum_{i=1}^{n} (R_A - \overline{R}_A)(R_B - \overline{R}_B)}{n-1}$<br>$\rho_{AB} = \frac{COV_{AB}}{\sigma_A \sigma_B}$ |
| **Risk consideration** | An assets stand alone risk considers the total risk of an asset. | A portfolio of asset considers the diversifiable risk only. |

## 2.6 CHOOSING THE OPTIMAL PORTFOLIO

### 2.6.1 The Efficient Frontier

The efficient frontier represents that set of portfolios that has the maximum rate of return for every given level of risk, or the minimum risk for every level of return. Every portfolio that lies on the efficient frontier has either a higher rate of return for equal risk or lower risk for an equal rate of return than some portfolio beneath the frontier. It is some time referred as the global mean variance portfolio.

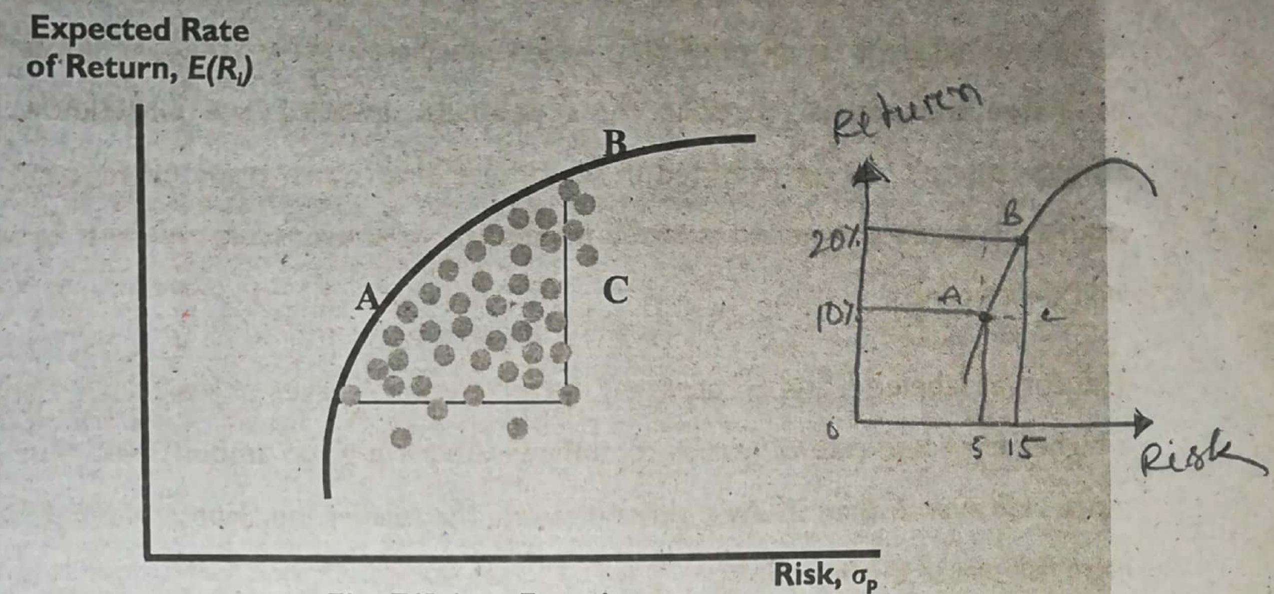



Fig: Efficient Frontier

Portfolios to the left of the efficient frontier are not possible because they lie outside the attainable set. Portfolios to the right of the efficient frontier are inefficient because some other portfolio would provide either a higher return with the same degree of risk or a lower risk for the same rate of return. In the above figure portfolio A dominates portfolio C because it has an equal rate of return but substantial less risk. Similarly, portfolio B dominates portfolio C because it has an equal risk but a higher expected rate of return. Indeed, portfolio C is dominated by all portfolios along curve AB because each one has the same or less risk while providing a greater or equal return.

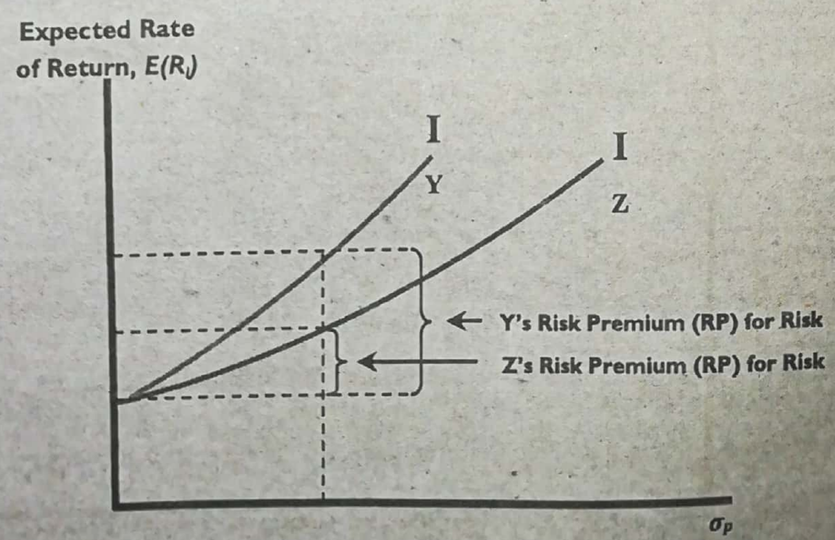



*Figure: Indifference Curves for Risk and Expected Return*

### 2.6.2 Risk/Return Indifference Curves

Given the efficient set of portfolios, which specific portfolio should an investor choose? To determine the optimal portfolio for a particular investor, we must know the investor's attitude toward risk as reflected in his or her risk/return trade-off function. An investor's risk/return tradeoff function is based on the standard economic concepts of *utility theory* and *indifference curves.*

The curves labeled $I_Y$ and $I_Z$ represent the indifference curves of individuals *Y* and Z. *Y* requires a higher expected rate of return to compensate for a given amount risk; thus, *Y* is said to be more risk averse than Z. As a generalization, *the steeper the slope of the indifference curve, the more risk averse the investor is.*

***The Optimal Portfolio for an Investor:*** The optimal portfolio is the portfolio on the efficient frontier that has the highest utility for a given investor. It lies at the point of tangency between the efficient frontier and the curve with the highest possible utility. This tangency point marks the highest level of satisfaction the investor can attain. A conservative investor's (Y) highest utility is at point *P*, where the curve $I_{Y2}$ just touches the efficient frontier. A less-risk-averse investor's (Z) highest utility occurs at point *Q*, which represents a portfolio with a higher expected return and higher risk than the portfolio at *P*.

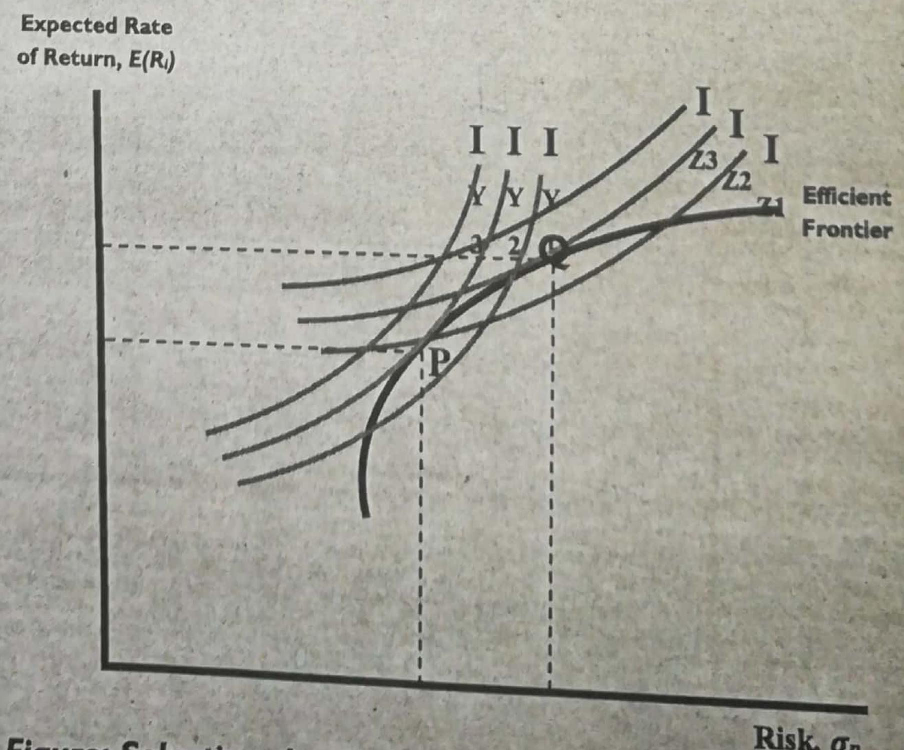



**Figure: Selecting the Optimal Portfolio of Risky Assets**

### 2.6.3 *How are total risk, non-diversifiable risk, and diversifiable risks related?*

Combining securities that are not perfectly correlated helps to lessen the risk of a portfolio. How much risk reduction is reasonable to expect, and how many different security holdings in a portfolio would be required? The following figure helps provide answers.

Research studies have looked at what happens to portfolio risk as randomly selected stocks are combined to form equally weighted portfolios. When we begin with a single stock, the risk of the portfolio is the standard deviation of that one stock. As the number of randomly selected stocks held in the portfolio is increased, the total risk of the portfolio is reduced. Such a reduction is at a decreasing rate, however. Thus a substantial proportion of the portfolio risk can be eliminated with a relatively moderate amount of diversification.

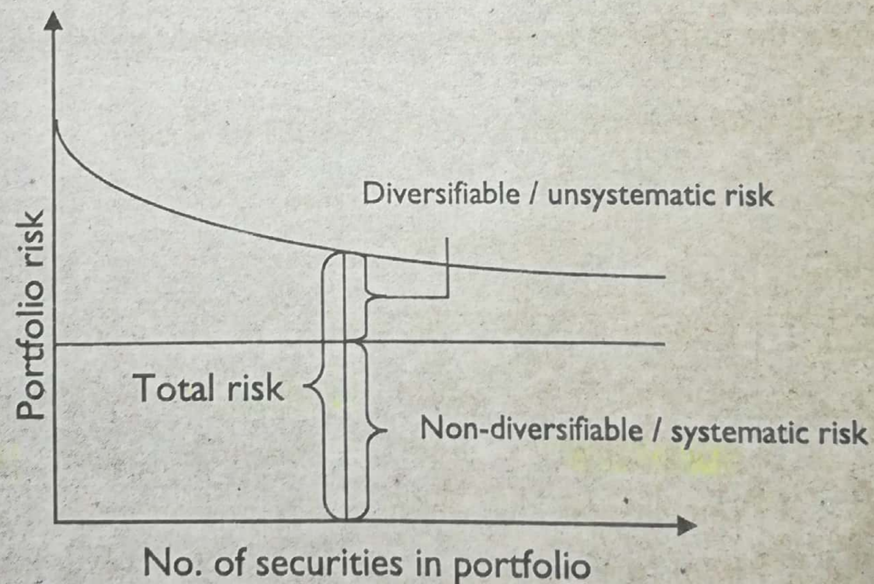



**Figure: Effect of Portfolio Size on Portfolio Risk**

As the figure shows, total portfolio risk comprises two components:

| | | | |
|---|---|---|---|
| *Total risk =* | *Systematic risk* (*non-diversifiable*) | + | *Unsystematic risk* (*diversifiable or avoidable*) |

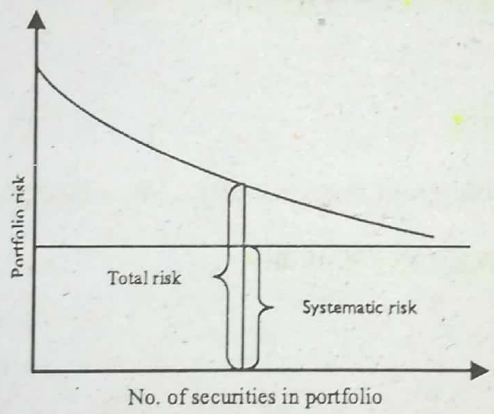
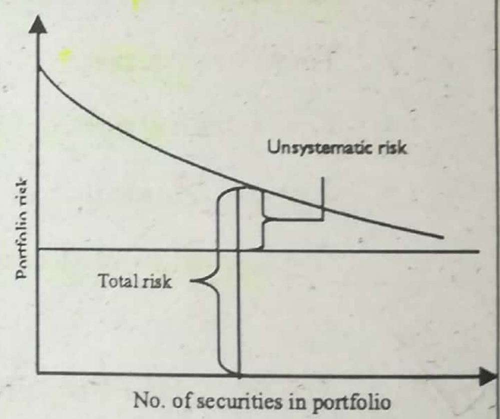
***Systematic Risk (Non-diversifiable):*** Systematic risk is due to risk factors that affect the overall market – such as changes in the nation's economy, tax reform, or a change in the world energy situation. These are risks that affect securities overall and, consequently, cannot be diversified away. In other words, even an investor who holds a well-diversified portfolio will be exposed to this type of risk.

***Unsystematic Risk (Diversifiable):*** Unsystematic risk is risk unique to a particular company or industry; it is independent of economic, political, and other factors that affect all securities in a systematic manner. A wildcat strike may affect only one company; a new competitor may begin to produce essentially the same product; or a technological breakthrough may make an existing product obsolete. However, by diversification this kind of risk can be reduced and even eliminated if diversification is efficient.

Therefore, not all of the risk involved in holding a stock is relevant, because part of this risk can be diversified away. The important risk of a stock is its unavoidable or systematic risk. Investors can expect to be compensated for bearing this systematic risk. They should not, however, expect the market to provide any extra compensation for bearing avoidable risk.

## 2.7 SYSTEMATIC RISK VS UNSYSTEMATIC RISK

| Point of Distinction | Systematic risk | Unsystematic risk |
|---|---|---|
| **Definition** | Systematic risk is that portion of the security risk which cannot be diversifiable from portfolio combination and which usually arises from the movement of market or economic forces. | Unsystematic risk is that portion of the security risk which can be diversifiable through portfolio formation and which usually arises from the movement of company specific factors. |
| **Factors** | Systematic risk is affected by macro economic factors such as variability of inflation, change in interest rate, change in money supply etc. | Unsystematic risk is affected by company specific factors such as wrong strategic planning, labor unrest, shortage of working capital, technological obsolescence. |

| Point of Distinction | Systematic risk | Unsystematic risk |
|---|---|---|
| Measurement | Systematic risk is measured by beta 'β' | Unsystematic risk is measured by error term epsilon 'ε' |
| Equation | The equation of systematic risk is= $\beta_m$ | Unsystematic risk is = 1 – systematic risk |
| Diversification | Cannot be diversified | Can be diversified |
| Return expectation | As it cannot be diversified investors require a premium or return for this risk. | Investors don't expect return against unsystematic risk because it can be diversified away. |
| Graph |   |   |
| Synonims | Systematic risk, also known as "market risk" or "un-diversifiable risk", is the uncertainty inherent to the entire market or entire market segment, may also referred to as market volatility. | Unsystematic risk, also known as "specific risk," "diversifiable risk" or "residual risk," is the type of uncertainty that comes with the company or industry you invest in. |

## 2.8 CAPITAL ASSET PRICING MODEL (CAPM)

In market equilibrium, a security is supposed to provide an expected return commensurate with its systematic risk – the risk that cannot be avoided by diversification. The greater the systematic risk of a security, the greater the return that investors will expect from the security. The relationship between beta (systematic risk) and expected return is known as the

Capital asset Pricing Model (CAPM). In the mid-1960s three economists William Sharpe, John Lintner, and Jack Treynor has developed this model, and it has had important implications for finance ever since.

As with any model, there are assumptions to be made.

- All investors are efficient investors, who want to target points on the efficient frontier.
- All investors are price takers.
- Investors can borrow or lend any amount of money at the risk-free rate of return.
- All investors have homogeneous expectations.
- All investors have the same one-period time horizon such as one month, six months, or one year.
- All investments are infinitely divisible and perfectly liquid.
- There are no taxes
- There are no transaction costs.
- There is no inflation or any change in interest rates, or inflation is fully anticipated.
- The quantities of all assets are given and fixed.

## 2.9 BETA COEFFICIENT

The beta coefficient, $\beta$, is a relative measure of non-diversifiable risk. It is an index of the degree of movement of an asset's return in response to a change in the market return. An asset's historical returns are used in finding the asset's beta coefficient. The market return is the return on the market portfolio of all traded securities. The *DSE All Share Price Index (DSI)*, *DSE General Index (DGEN)* or some similar stock index is commonly used as the market return.

***Deriving Beta from Return Data:*** An asset's historical returns are used in finding the asset's beta coefficient. Figure 6 plots the relationship between the returns of two assets – R and S – and the market return. Note that the horizontal (x) axis measures the historical market returns and the vertical (y) axis measures the individual asset's historical returns. The first step in deriving beta involves plotting the coordinates for the market return and asset returns from various points in time. By use of statistical techniques, the 'characteristic line' that best explains the relationship between the asset return and the market return coordinates is fit to the data points. The slope of this line is *beta*. Mathematically,

$$\beta_i = \frac{COV_{i,m}}{\sigma_m^2} = \frac{\sigma_i \sigma_m \rho_{im}}{\sigma_m^2}$$

The beta for asset *R* is 0.763 and that for asset *S* is 1.127. Asset *S*'s higher beta (steeper characteristic line slope) indicates that its return is more responsive to changing market returns. *Therefore asset S is more risky than asset R.*

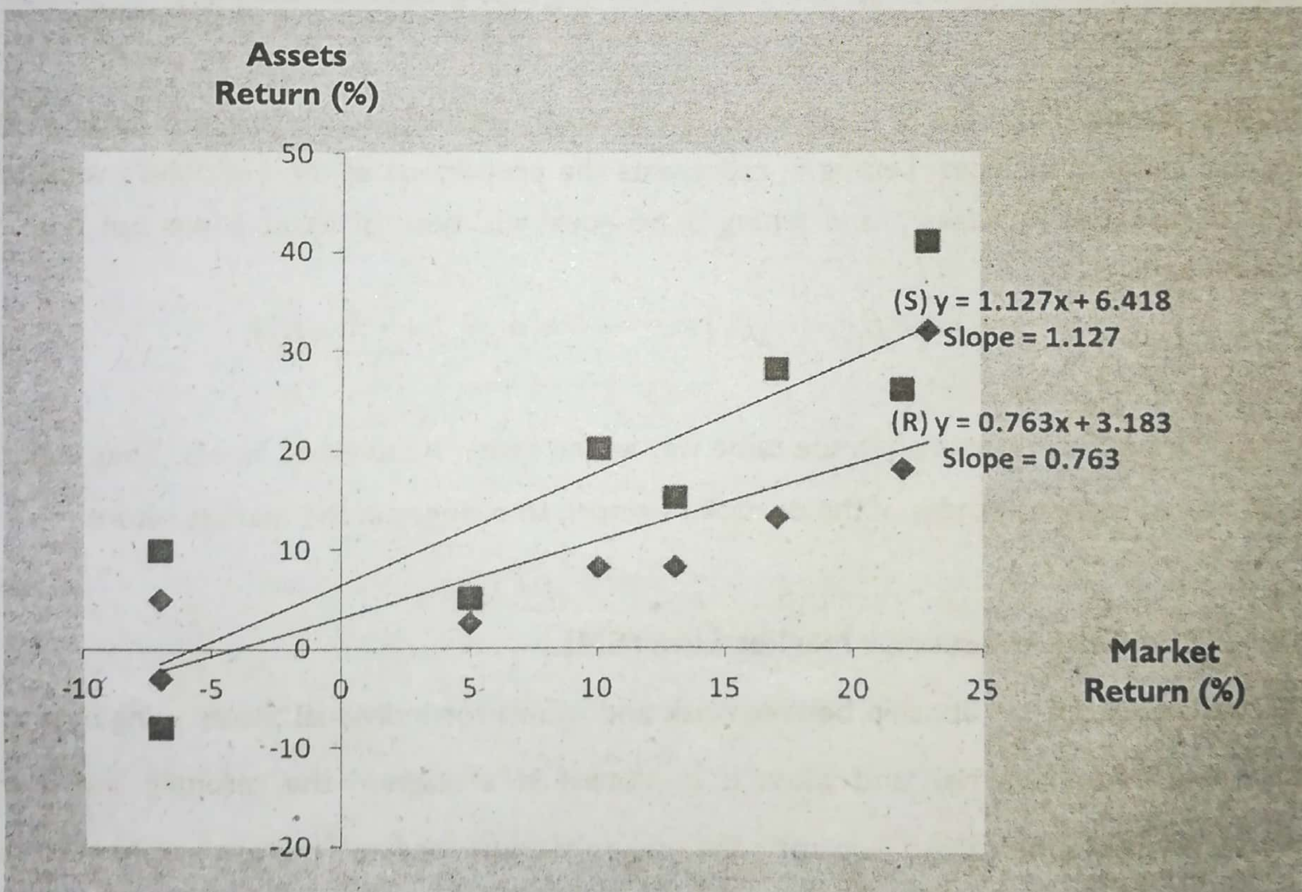



***Figure: Characteristic Line of two Assets S,R***

***Interpreting Betas:*** The beta coefficient for the market is considered to be equal to 1.0. All other betas are viewed in relation to this value. Asset betas may be positive or negative, but positive betas are the norm. The majority of beta coefficients fall between 0.5 and 2.0. The return of a stock that is half as responsive as the market ($\beta$=0.5) is expected to change by ½ percent for each 1 percent change in the return of the market portfolio. A stock that is twice as responsive as the market ($\beta$=2.0) is expected to experience a 2 percent change in its return for each 1 percent change in the return of the market portfolio. The following table provides various beta values and their interpretations.

TABLE **Selected Beta Coefficients and Their Interpretations**

| Beta | Comment | Interpretation |
|---|---|---|
| 2.0 | Move in same direction as market | Twice as responsive as the market |
| 1.0 | | Same response as the market |
| 0.5 | | Only half as responsive as the market |
| 0 | | Unaffected by market movement |
| -0.5 | Move in opposite direction to market | Only half as responsive as the market |
| -1.0 | | Same response as the market |
| -2.0 | | Twice as responsive as the market |

***Portfolio Betas:*** The beta of a portfolio can be easily estimated by using the betas of the individual assets it includes. Letting $w_j$ represents the proportion of the portfolio's total taka value represented by asset $j$, and letting $\beta_j$ be equal the beta of asset $j$, we can find the portfolio beta, $\beta_p$

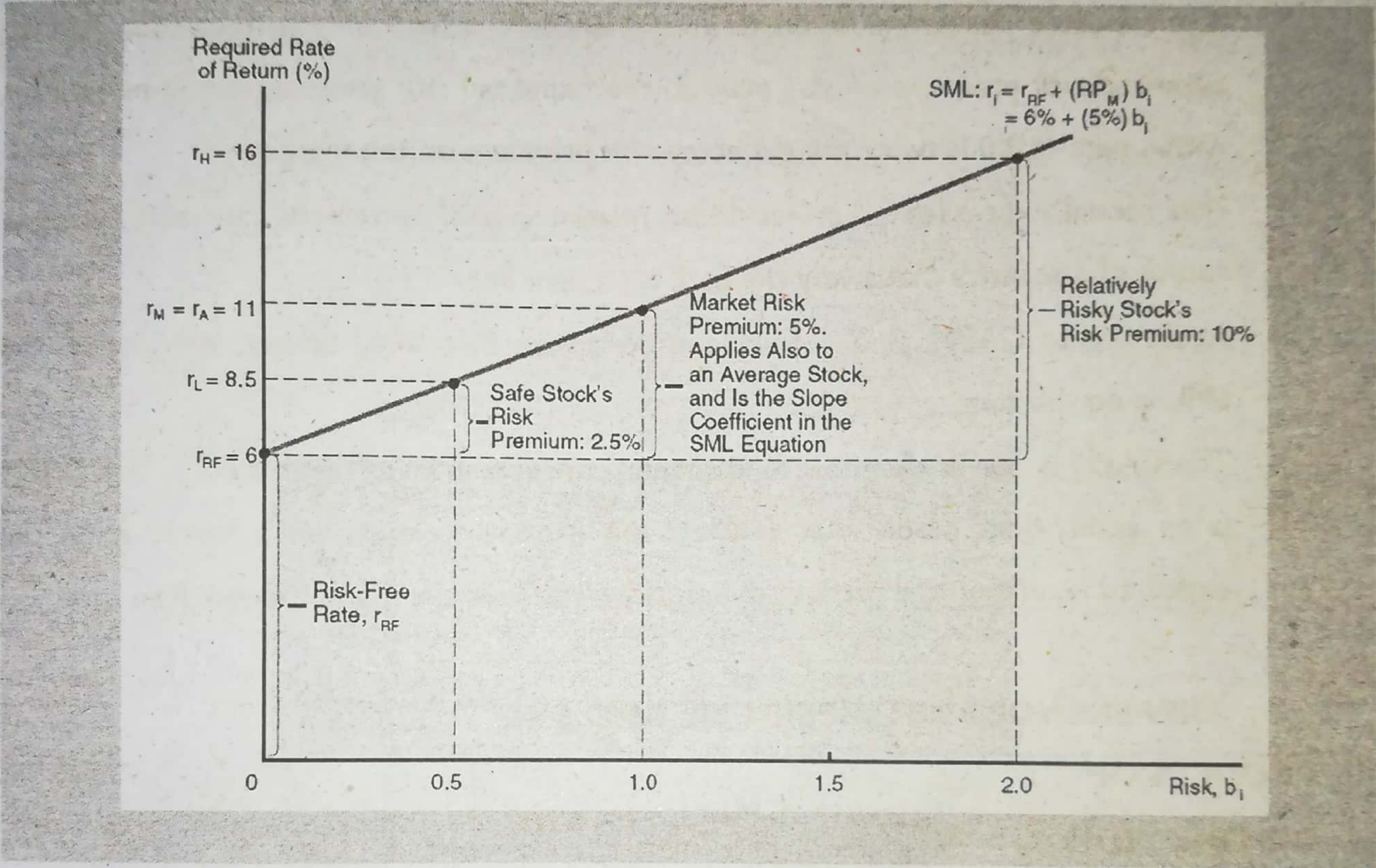
$$\beta_p = (w_1\beta_1) + (w_2\beta_2) + ----+(w_n\beta_n) = \sum_{i=1}^{n}(w_j\beta_j)$$

Portfolio betas are interpreted in the same way as the betas of individual assets. They indicate the degree of responsiveness of the portfolio's return to changes in the market return.

## 2.10 The Graph; Security Market Line (SML)

SML represents the relationship between risk and return for individual assets using beta as a measure of systematic risk and when it is plotted in a diagram the resulting line is the ***Security Market Line***. Wise investors don't take risks just for fun. They are playing with real money. Therefore, they require a higher return from the market portfolio than from Treasury bills. The difference between the return on the market and the interest rate is termed the market risk premium. The market portfolio has a beta of 1.0 and a risk premium of ($r_m - r_f$). This gives us two benchmarks for the expected risk premium. But what is the expected risk premium when beta is not 0 or 1? In the mid-1960s three economists—William Sharpe, John Lintner, and Jack Treynor—produced an answer to this question. Their answer is known as the ***Capital Asset Pricing Model***, or **CAPM**.

*Let's assume that $r_m$ is 11% and $r_f$ is 6%.* As we know the market beta is always 1, so the risk premium against the systematic risk is (11-6) = 5%. But the risk premium directly varies with the change in the security beta.




The model's message is both startling and simple. In a competitive market, the expected risk premium varies in direct proportion to beta. This means that all investments must plot along the sloping line, known as the ***Security Market Line (SML).*** The expected risk premium on an investment with a beta of .5 is, therefore, half the expected risk premium on the market; the expected risk premium on an investment with a beta of 2.0 is twice the expected risk premium on the market.

### 2.10.1 Significance of Security Market Line

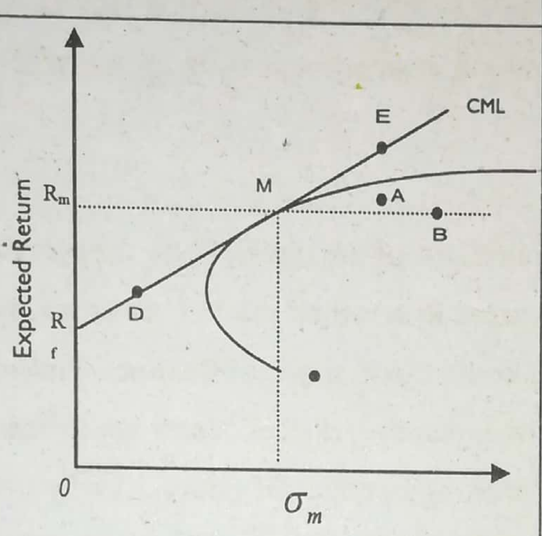
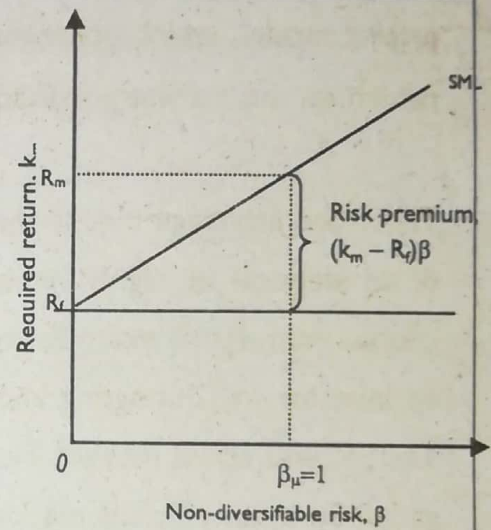
The model's message is both startling and simple;

- In a competitive market, the expected risk premium varies in direct proportion to beta. This means that all investments must plot along the sloping line, known as the ***Security Market Line (SML).***

- SML uses beta on the X-axis, so in CAPM world all properly priced securities and portfolios of securities will plot on the SML.
- The expected risk premium on an investment with a beta of .5 is, therefore, half the expected risk premium on the market; the expected risk premium on an investment with a beta of 2.0 is twice the expected risk premium on the market.
- This model expresses a perfect linear relationship of systematic risk and expected return of a security that's why the SML is straight line.
- According to CAPM, all securities and portfolios, diversified or not, will plot on the SML in equilibrium.
- The CAPM is one of the most fundamental concepts in investment theory. The CAPM is an equilibrium model that predicts the expected return on a stock, given the expected return on the market, the stock's beta coefficient, and the risk free rate.

## 2.11 DIFFERENCES BETWEEN CML AND SML

| POINTS OF DISTINCTION | CML | SML |
|---|---|---|
| **Definition** | The introduction of a risk free asset changes the Markowitz efficient frontier into a straight line called ***Capital Market Line*** . or CML is the graphical presentation of equilibrium relationship between expected return and total risk for efficiently diversified portfolios. | SML represents the relationship between risk and return for individual assets using beta as a measure of systematic risk and when it is plotted in a diagram the resulting line is the ***Security Market Line*** |
| **Equation** | $E(R_p)=RF+\sigma_p\left\{\frac{[E(R_m)-RF]}{\sigma_m}\right\}$ | $R_i=R_f+\beta_i(R_m-R_f)$ |
| **Risk Measurement** | CML uses total risk on the X-axis , hence only efficient portfolios will | SML uses beta on the X-axis, so in CAPM world all properly |

| POINTS OF DISTINCTION | CML | SML |
|---|---|---|
| | plot on the CML | priced securities and portfolios of securities will plot on the SML. |
| **Figure** |   |   |
| **Slope** | $\text{Slope of CML} = \sigma_p \left\{ \frac{[E(R_m) - RF]}{\sigma_m} \right\}$ | $\textit{Slope of SML} = (R_m - R_f)$ |
| **Risk Consideration** | CML considers both systematic and unsystematic risk. | SML considers only systematic risk. |

## 2.12 ARBITRAGE PRICING THEORY (APT) ; AN ALTERNATIVE TO THE CAPM

Like the capital asset pricing model, arbitrage pricing theory stresses that expected return depends on the risk stemming from economy wide influences and is not affected by unique risk. You can think of the factors in arbitrage pricing as representing special portfolios of stocks that tend to be subject to a common influence. If the expected risk premium on each of these portfolios is proportional to the portfolio's market beta, then the arbitrage pricing theory and the capital asset pricing model will give the same answer. In any other case they won't. How do the two theories stack up? Arbitrage pricing has some attractive features. For example, the market portfolio that plays such a central role in the capital asset pricing model does not feature in arbitrage pricing theory. So we don't have to worry about the problem of measuring the market portfolio, and in principle we can test the arbitrage pricing theory even if we have data on only a sample of risky assets. Unfortunately you win some and lose some. Arbitrage pricing theory doesn't tell us what the underlying factors are unlike the capital asset pricing model, which collapses *all* macroeconomic risks into a well-defined *single* factor, the return on the market portfolio.

The word arbitrage means the purchase of an asset in one market and the simultaneous sale of an identical or highly similar asset in another market so as to make a profit on the two transactions. If, for example, you could buy a share of Beximco Pharma stock for Tk.153 from an investor in Chittagong and immediately sell that share to another investor in Dhaka for Tk.155, you could make a Tk.2 arbitrage profit. Of course, everyone would like to play that game. So in an efficient market, which is characterized by a great deal of competition among investors, the arbitrage process will cause highly similar assets to have equal prices. That is, if the prices of highly similar assets were not equal, the arbitrage process would work until price equality occurred. In the example given above, shares of Beximco Pharma, being economically indistinguishable from each other, should all sell for the same price regardless of which region the buyer of seller lives in.

Stated differently, the arbitrage process leads to the *law of one price*, which says that equivalent assets should be priced equally. This law also implies that equivalent assets should have equal expected and required rate of return. Using this arbitrage concept, a risk-return model has been developed that is today challenging the CAPM as a theoretical and empirical foundation of finance. Recall from previous equation that the CAPM relates an asset's required rate of return with the asset's risk (beta), the risk-free interest rate ($R_f$), and the market portfolio's rate of return ($R_m$):

$$r_i = r_f + \beta_i (r_m - r_f)$$

Viewed abstractly, this equation can be rewritten as:

$$r_i = r_f + \beta_i F_M$$

Where,

$F_M$ = market risk premium = market factor = $(r_m - r_f)$

Recall that $\beta_i$ measures the risk of asset *I* relative to the market index. That is, the CAPM views asset *I*'s risk as being entirely market related and equal to a single beta value, $\beta_i$. Essentially, the arbitrage pricing theory views the CAPM as being too simple in assuming an asset's risk is described completely by a single beta that is related to the market index. Rather, the APT views asset risk as better portrayed by relating an asset's returns with several indices. That is, an asset has several relevant betas, not just one.

A simple version of the APT is:

$$R_i = R_f + \beta_{i\,1}F_1 + \beta_{i\,2}F_2 + \beta_{i\,3}F_3$$

Where,

$F_1, F_2, F_3$ = excess returns on broad factors that are important determinants of asset rates of return

$\beta_1, \beta_2, \beta_3$ = risk of asset *I* relative to each of the three factors, respectively

In this simple version of the APT, three broad factors affect risk and return. Therefore, three sensitivity coefficients (beta) convert these factors into a required rate of return for any given asset.

The APT is thus a more complex theory because it allows for more risk elements for asset *I* than only the market-related risk. Exactly what $F_I$, $F_2$, and $F_3$ and other factors economically stand for is unresolved today and is the subject of ongoing research. Some researchers have suggested unanticipated movements in inflation, industrial production, and the general cost of risk bearing. Other suggestions have leaned more toward a national economy factor and industry factors.

Proponents of the APT model argue that it is more general than the CAPM because the CAPM is a single-factor model but the APT is a multifactor model. APT proponents also say that the CAPM has not performed well in statistical verification tests. On the other side, testing of the validity of the APT has been performed only recently and remains unfinished. Evidence supporting the APT as being a better risk-return model than the CAPM is inconclusive and hotly debated at this time. It also bothers many finance scholars that the APT fails to state explicitly how many factors are important and exactly what those factors are. It's also too early to tell if the APT model will eventually replace the CAPM as the main theoretical model in finance. We will take the position that the CAPM is the best model we have today.

### 2.12.1 APT an Example

Arbitrage pricing theory will provide a good handle on expected returns only if we can (1) identify a reasonably short list of macroeconomic factors,25 (2) measure the expected risk premium on each of these factors, and (3) measure the sensitivity of each stock to these factors. Let us look briefly at how Elton, Gruber, and Mei tackled each of these issues and estimated the cost of equity for a group of nine New York utilities.

***Step 1: Identify the Macroeconomic Factors*** Although APT doesn't tell us what the underlying economic factors are, Elton, Gruber, and Mei identified five principal factors that could affect either the cash flows themselves or the rate at which they are discounted. These factors are;

| *Factor* | *Measured by* |
|---|---|
| Yield spread | Return on long government bond *less* return on 30-day Treasury bills |
| Interest rate | Change in Treasury bill return |
| Exchange rate | Change in value of dollar relative to basket of currencies |
| Real GNP | Change in forecasts of real GNP |
| Inflation | Change in forecasts of inflation |

| Factor | Estimated Risk Premium* $(r_{factor} - r_f)$ |
|---|---|
| Yield spread | 5.1% |
| Interest rate | -.61 |
| Exchange rate | -.59 |
| Real GNP | .49 |
| Inflation | -.83 |
| Market | 6.36 |

**Estimated risk premiums for taking on factor risks, 1978–1990.**

*The risk premiums have been scaled to represent the annual premiums or the average industrial stock in the Elton–Gruber–Mei sample.

Source: E. J. Elton, M. J. Gruber, and J. Mei, "Cost of Capital Using Arbitrage Pricing Theory: A Case Study of Nine New York Utilities," *Financial Markets, Institutions, and Instruments* 3 (August 1994), pp. 46–73.

*Table: Estimated Risk Premium*

To capture any remaining pervasive influences, Elton, Gruber, and Mei also included a sixth factor, the portion of the market return that could not be explained by the first five.

***Step 2: Estimate the Risk Premium for Each Factor*** some stocks are more exposed than others to a particular factor. So we can estimate the sensitivity of a sample of stocks to each factor and then measure how much extra return investors would have received in the past for taking on factor risk. The results are shown in the estimated risk premium table.

For example, stocks with positive sensitivity to real GNP tended to have higher returns when real GNP increased. A stock with an average sensitivity gave investors an additional return of .49 percent a year compared with a stock that was completely unaffected by changes in real GNP. In other words, investors appeared to dislike "cyclical" stocks, whose returns were sensitive to economic activity, and demanded a higher return from these stocks. By contrast, estimated risk premium table shows that a stock with average exposure to *inflation* gave investors .83 percent a year *less* return than a stock with no exposure to inflation. Thus investors seemed to prefer stocks that protected them against inflation (stocks that did well when inflation accelerated), and they were willing to accept a lower expected return from such stocks.

**Step 3: Estimate the Factor Sensitivities** The estimates of the premiums for taking on factor risk can now be used to estimate the cost of equity for the group of New York State utilities. Remember, APT states that the risk premium for any asset depends on its sensitivities to factor risks ($b$) and the expected risk premium for each factor $(r_{factor} - r_f)$. In this case there are six factors,

$$r - r_f = b_1(r_{factor1} - r_f) + b_2(r_{factor2} - r_f) + \ldots\ldots\ldots\ldots + b_6(r_{factor6} - r_f)$$

So the first column of ***expected risk premium table*** shows the factor risks for the portfolio of utilities, and the second column shows the required risk premium for each factor taken from ***estimated risk premium table.*** The third column is simply the product of these two numbers. It shows how much return investors demanded for taking on each factor risk. To find the expected risk premium, just add the figures in the final column:

$$\text{Expected risk premium} = (r - r_f) = 8.53\%$$

The one-year Treasury bill rate in December 1990, the end of the Elton–Gruber–Mei sample period, was about 7 percent, so the APT estimate of the expected return on New York State utility stocks was;

$$\text{Expected Return} = \text{Risk-free interest} + \text{expected risk premium}$$
$$= 7 + 8.53 = 15.53 \text{ or } 15.5\%$$

| Factor | Factor Risk $b_i$ | Estimated Risk Premium $(r_{factor} - r_f)$ | Factor Risk Premium $b(r_{factor} - r_f)$ |
|---|---|---|---|
| Yield spread | 1.04 | 5.1% | 5.30% |
| Interest rate | -2.25 | -.61 | 1.37 |
| Exchange rate | .70 | -.59 | -.41 |
| Real GNP | .17 | .49 | .08 |
| Inflation | -.18 | -.83 | .15 |
| Market | .32 | 6.36 | 2.04 |
| Total | | | 8.53% |

*Table: Using APT to estimate the expected risk premium for a portfolio of nine New York State utility stocks.*

## 2.13 CAPM VS. APT

| POINT OF DISTINCTION | CAPM | APT |
|---|---|---|
| **Definition** | CAPM is based on investors' portfolio demand and equilibrium arguments. | APT is based on the factor model of returns and the approximate arbitrage argument. |
| **Factor Consideration** | CAPM relates an asset's required rate of return with the asset's risk (beta), the risk-free interest rate ($R_f$) and the market portfolio's rate of return ($R_m$). The simple version of CAPM is : $r_i = r_f + \beta_i (r_m - r_f)$ | APT views asset risk as better portrayed by relating an asset's returns with several indices. That is, an asset has several relevant betas, not just one. A simple version of APT is: $r_i = r_f + \beta_{i1} F_1 + \beta_{i2} F_2 + \beta_{i3} F_3$ |

| POINT OF DISTINCTION | CAPM | APT |
|---|---|---|
| **Methodology** | CAPM is an equilibrium model and derived from individual portfolio optimization. | APT is a statistical model which tries to capture sources of systematic risk. Here relation between sources is determined by no arbitrage condition. |
| **Application** | In CAPM it is difficult to find good proxy for market return. | In APT it is difficult to identify appropriate factors. |
| **Assumption regarding probability distribution of asset return** | CAPM assumes that the probability distributions of asset returns are normally distributed. | APT does not make any assumption about the distribution of asset returns. |
| **Utility function** | According to CAPM investors are all risk-averse who maximize their expected utility of their end of period wealth. | APT does not make any strong assumption about utility function (only risk-averse). |
| **Role of the market portfolio** | CAPM requires that the market portfolio be efficient. | There is no special role of the market portfolio in APT. |

SELF-TEST | REVIEW QUESTIONS | Concept Checkers

**Q - 2.8** What is a portfolio? How is the portfolio return and risk calculated for a two security portfolio? (NU BBA – 2008)

**Q - 2.9** Differenciate between portfolio risk and stand alone risk. (NU BBA – 2008)

**Q - 2.10** What is efficient frontier? How does an Investor choose his or her Optimal Portfolio from an Efficient Set? (NU BBA – 2007, 2008, 2009, 2011, 2012 )

**Q - 2.11** How are total risk, non-diversifiable risk, and diversifiable risks related?

**Q - 2.12** What is CAPM? Explain its assumptions. (NU BBA – 2009, 2010, 2013)

**Q - 2.13** Explain the Security Market Line with the help of a figure.

Or, Why is the SML a straight line? (NU BBA – 2008)

**Q - 2.14** Show the differences between CML and SML. (NU BBA – 2008, 2012)

**Q - 2.15** What is beta? How is it measured?

**Q - 2.16** Define and distinguish between systematic risk and unsystematic risk. (NU BBA 2007)

**Q - 2.17** Discuss the process of calculating expected return using APT.

**Q - 2.18** What is the significance of Security Market Line (SML)? (NU BBA 2009)

**Q-2.19** Define and distinguish between CAPM and APT. (NU BBA 2007, 2008, 2009,

**2-1: (NU BBA – 2007)** Black Berry Corporation is considering three possible capital projects for the next year. Each project has a 1-year life and project returns depend on next year's state of the economy. The estimated rates of return are shown in the table:

| MARKET CONDITION | PROBABILITY | RATES OF RETURN IF STATE OCCURS | | |
|---|---|---|---|---|
| | | A | B | C |
| Pessimistic | 0.25 | 20% | 18% | 28% |
| Most likely | 0.50 | 28 | 26 | 24 |
| Optimistic | 0.25 | 32 | 36 | 20 |

Also Assume that Black Berry Corporation is going to invest one third of its available funds in each project. Black Berry will create a portfolio of three equally weighted projects.

**Requirements:**

1. Find each project's expected rate of return, variance, and standard deviation.
2. What is the expected rate of return, variance, and standard deviation of the three-projects' portfolio?

**Solution:**

***I. Calculation of expected rate of return:***

$$\bar{x}_A = (.25\times.20)+(.50\times.28)+(.25\times.32) = 0.27$$

$$\bar{x}_B = (.25\times.18)+(.50\times.26)+(.25\times.36) = 0.265$$

$$\bar{x}_c = (.25\times.28)+(.50\times.24)+(.25\times.20) = 0.24$$

***Calculation of standard deviation:***

$$\sigma_A = \sqrt{.25(.20-.27)^2+.50(.28-.27)^2+.25(.32-.27)^2}$$
$$= \sqrt{0.0019} = .0435$$

$$\sigma_B = \sqrt{.25(.18-.265)^2+.50(.26-.265)^2+.25(.36-.265)^2}$$
$$= \sqrt{0.004075} = .0638$$

$$\sigma_C = \sqrt{.25(.28-.24)^2+.50(.24-.24)^2+.25(.20-.24)^2}$$
$$= \sqrt{0.0008} = .02828$$

**Calculation of Variance:**

$$\sigma_A^2 = (0.0435)^2 = 0.0019$$

$$\sigma_B^2 = (0.0638)^2 = 0.0041$$

$$\sigma_C^2 = (0.02828)^2 = 0.0008$$

II. Portfolio return, $\bar{x}_P = \sum_{i=1}^{n} w_i \bar{x}_i. = (.333 \times .27) + (.333 \times 0.265) + (.333 \times .24) = .258075$

Portfolio standard deviation,

$$\sigma_P = \sqrt{w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + 2w_A w_B COV_{AB} + 2w_B w_C COV_{BC} + 2w_C w_A COV_{CA}}$$

$$\sigma_p = \sqrt{\begin{matrix}(.333)^2(.0435)^2 + (.333)^2(.0638)^2 + (.333)^2(.02828)^2 + (2\times.00265\times.333\times333) \\ +(2\times-0.0012\times.333\times.333) + (2\times-.0018\times.333\times.333)\end{matrix}}$$

$$= \sqrt{0.000672} = .0259$$

Variance, $\sigma_P^2 = (0.259)^2 = 0.00067$

Covariance, $COV_{AB} = \sum_{i=1}^{n} p_i (x_A - \bar{x}_A)(x_B - \bar{x}_B).$

$$COV_{AB} = .25(.20-.27)(.18-.265) + .5(.28-.27)(.26-.265) + .25(.32-.27)(.36-.265) = .00265$$

$$COV_{CA} = .25(.20-.27)(.28-.24) + .5(.28-.27)(.24-.24) + .25(.32-.27)(.20-.24) = -.0012$$

$$COV_{BC} = .25(.18-.265)(.28-.24) + .5(.26-265.)(.24-.24) + .25(.36-.265)(.20-.24) = -.0018$$

**2-2: (NU BBA – 2008)** Mr. Henry can invest in Highbull stock and Slowbear stock. His projection of the returns on these two stocks is as follows:-

| State of the Economy | Probability of State Occurring | Return on Highbull Stock (%) | Return on Slowbear Stock (%) |
|---|---|---|---|
| Recession | .25 | -2.00 | 5.00 |
| Normal | .60 | 9.20 | 6.20 |
| Boom | .15 | 15.40 | 7.40 |

(i) Calculate the expected return on each stock.

(ii) Calculate the standard deviation of returns on each stock.

(iii) Calculate the covariance and correlation between returns on the two stocks.

**Solution:**

I. Calculation of expected rate of return: Let's assume Highbull = H and Slowbear = S

$$\bar{x}_H = (.25 \times -.02) + (.60 \times .092) + (.15 \times .1540) = 0.0733$$

$$\bar{x}_S = (.25 \times .05) + (.60 \times .0620) + (.15 \times .0740) = 0.0608$$

**Calculation of standard deviation:**

$$\sigma_H = \sqrt{.25(-.02 - .0733)^2 + .60(.0920 - .0733)^2 + .15(.1540 - .0733)^2}$$
$$= \sqrt{0.00336291} = .058$$

$$\sigma_S = \sqrt{.25(.05 - .0608)^2 + .60(.0620 - .0608)^2 + .15(.0740 - .0608)^2}$$
$$= \sqrt{0.00005616} = .0075$$

***Covariance:***

$$COV_{HC} = .25(-.2 - .0733)(.05 - .0608) + .6(.0920 - .0733)(.0620 - .0608) +$$
$$15(.1540 - .0733)(.0740 - .0608) = .00042516$$

**Correlation:** $$\rho_{HS} = \frac{COV_{HS}}{\sigma_H \sigma_S} = \frac{0.00042516}{.058 \times .0075} = 0.9774$$

**2-3: *(NU BBA 2009, 2012)*** Security X and Y have the following characteristics: -

| Security X | | Security Y | |
|---|---|---|---|
| Return | Probability | Return | Probability |
| 30% | 0.10 | -20% | 0.05 |
| 20 | 0.20 | 10 | .25 |
| 10 | 0.40 | 20 | .30 |
| 5 | 0.20 | 30 | .30 |
| -10 | 0.10 | 40 | .10 |

You are required to calculate:

1. The expected return and standard deviation of return for each security;
2. The expected return and standard deviation of portfolio of X and Y combined with equal weights.

**Solution:**

***I. Calculation of expected rate of return:***

$$\bar{x}_X = (.10 \times .3) + (.20 \times .2) + (.40 \times .1) + (.20 \times .05) + (.10 \times .-.10) = .11$$

$$\bar{x}_Y = (.05 \times -.20) + (.25 \times .10) + (.30 \times .20) + (.30 \times .30) + (.10 \times .40) = .205$$

***Calculation of standard deviation:***

$$\sigma_x = \sqrt{.10(.30-.11)^2 + .20(.20-.11)^2 + .40(.10-.11)^2 + .20(.05-.11)^2 + .10(-.10-.11)^2}$$
$$= \sqrt{.0104} = .102$$

$$\sigma_y = \sqrt{.05(.-.20-.205)^2 + .25(.10-.205)^2 + .30(.20-.205)^2 + .30(.30-.205)^2 + .10(.40-.205)^2}$$
$$= \sqrt{.0175} = .1322$$

***II. Portfolio return,*** $\bar{x}_P = \sum_{i=1}^{n} w_i \bar{x}_i. = (.50 \times .11) + (.50 \times 0.205) = .1575$

**Portfolio standard deviation,** $\sigma_P = \sqrt{w_x^2 \sigma_x^2 + w_y^2 \sigma_y^2 + 2 w_x w_y COV_{AB}}$

$$\sigma_P = \sqrt{(.5)^2(.102)^2 + (.5)^2(.1322)^2 + 2 \times .5 \times .5 \times -.0016} = \sqrt{0.0062} = .0785$$

**Covariance,** $COV_{xy} = \sum_{i=1}^{n} p_{ij}(x_x - \bar{x}_x)(x_y - \bar{x}_y)$

$$COV_{xy} = .10 \times .05(.30-.11)(-.2-.205) + .20 \times .25(.20-.11)(.10-.205) + .40 \times .30(.10-.11)(.20-.205)$$
$$+ .20 \times .30(.05-.11)(.30-.205) + .10 \times .10(-.10-.11)(.40-.205) = -.0016$$

**2-4: (NU BBA – 2010)** Assume that the risk-free rate is 5% and market risk premium is 7%. What is the expected return for the overall stock market? What is the required rate of return on a stock that has a beta of 1.2?

**Solution:**

We know, Risk Premium = $(R_m - R_f)$, here Risk Premium = 7,

So, $R_m = R_f + 7 = 5 + 7 = 12$

Expected rate of return, $E(R_i) = R_f + (R_m - R_f)\beta$

$$E(R_i) = 5 + (7)1.2 = 13.4\%$$

**2-5: (NU BBA – 2010, 2014)** Stock X and Y have the following probability distribution of expected returns: -

| Probability | x | y |
|---|---|---|
| 0.1 | -10% | -35% |
| .20 | 2 | 0 |
| .40 | 12 | 20 |
| .20 | 20 | 25 |
| .10 | 38 | 45 |

(i) Calculate the expected return for both stocks.

(ii) Calculate the standard deviation for both stocks.

(iii) Which stock is risky and why?

**Solution:**

**I. Calculation of expected rate of return:**

$$\bar{x}_X = (.10 \times -.10) + (.20 \times .02) + (.40 \times .12) + (.20 \times .20) + (.10 \times .38) = .12$$

$$\bar{x}_Y = (.10 \times -.35) + (.20 \times .00) + (.40 \times .20) + (.20 \times .25) + (.10 \times .45) = .14$$

**Calculation of standard deviation:**

$$\sigma_x = \sqrt{.10(.-.10-.12)^2 + .20(.02-.12)^2 + .40(.12-.12)^2 + .20(.20-.12)^2 + .10(.38-.12)^2}$$
$$= \sqrt{0.01488} = .12198$$

$$\sigma_y = \sqrt{.10(-.35-.14)^2 + .20(.00-.14)^2 + .40(.20-.14)^2 + .20(.25-.14)^2 + .10(.45-.14)^2}$$
$$= \sqrt{0..0414} = .2035$$

iii.

$$CV_X = \frac{0.12198}{0.12} = 1.0165 \quad CV_y = \frac{0.2035}{0.14} = 1.45$$

Stock y is more risky considering the standard deviations of security x and y.

**2-6: (NU BBA – 2011)** Black Berry Corporation is considering three possible capital projects for the next year. Each project has a 1-year life and project returns depend on next year's state of the economy. The estimated rates of return are shown in the table:

| MARKET CONDITION | PROBABILITY | RATES OF RETURN IF STATE OCCURS | | |
|---|---|---|---|---|
| | | A | B | C |
| Pessimistic | 0.25 | 20% | 18% | 28% |
| Most likely | 0.50 | 28 | 26 | 24 |
| Optimistic | 0.25 | 32 | 36 | 20 |

Also Assume that Black Berry Corporation is going to invest one third of its available funds in each project. Black Berry will create a portfolio of three equally weighted projects.

**Requirements:**

1. Find each project's expected rate of return, variance, and standard deviation.
2. What is the expected rate of return, variance, and standard deviation of the three-projects' portfolio?

**Solution:**

***I. Calculation of expected rate of return:***

$$\overline{R}_A = (.25\times.20)+(.50\times.28)+(.25\times.32) = 0.27$$

$$\overline{R}_B = (.25\times.18)+(.50\times.26)+(.25\times.36) = 0.265$$

$$\overline{R}_c = (.25\times.28)+(.50\times.24)+(.25\times.20) = 0.24$$

**Calculation of standard deviation:**

$$\sigma_A = \sqrt{.25(.20-.27)^2+.50(.28-.27)^2+.25(.32-.27)^2}$$
$$=\sqrt{0.0019} = .0435$$

$$\sigma_B = \sqrt{.25(.18-.265)^2+.50(.26-.265)^2+.25(.36-.265)^2}$$
$$=\sqrt{0.004075} = .0638$$

$$\sigma_C = \sqrt{.25(.28-.24)^2+.50(.24-.24)^2+.25(.20-.24)^2}$$
$$=\sqrt{0.0008} = .02828$$

**Calculation of Variance:**

$$\sigma_A^2 = (0.0435)^2 = 0.0019$$

$$\sigma_B^2 = (0.0638)^2 = 0.0041$$

$$\sigma_C^2 = (0.02828)^2 = 0.0008$$

***II.*** **Portfolio return,**

Portfolio return, $\bar{x}_P = \sum_{i=1}^{n} w_i\bar{x}_i. = (.333\times.27)+(.333\times0.265)+(.333\times.24) = .258075$

***Portfolio standard deviation,***

$$\sigma_P = \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + 2w_Aw_BCOV_{AB} + 2w_Bw_CCOV_{BC} + 2w_Cw_ACOV_{CA}}$$

$$= \sqrt{(.333)^2(.0435)^2 + (.333)^2(.0638)^2 + (.333)^2(.02828)^2 + (.00265 \times .333 \times 333) + (-0.0012 \times 2 \times .333 \times .333) + (-.0018 \times 2 \times .333 \times .333)}$$

$$= \sqrt{0.000672} = .0259$$

**Variance,** $\sigma_P{}^2 = (0.259)^2 = 0.00067$

**Covariance,** $COV_{AB} = \sum_{i=1}^{n} p_i (R_A - \bar{R}_A)(R_B - \bar{R}_B)$

$$COV_{AB} = .25(.20-.27)(.18-.265) + .5(.28-.27)(.26-.265) + .25(.32-.27)(.36-.265)$$
$$= .00265$$

$$COV_{CA} = .25(.20-.27)(.28-.24) + .5(.28-.27)(.24-.24) + .25(.32-.27)(.20-.24)$$
$$= -.0012$$

$$COV_{BC} = .25(.18-.265)(.28-.24) + .5(.26-265.)(.24-.24) + .25(.36-.265)(.20-.24)$$
$$= -.0018$$

**2-8: (NU BBA – 2013)** If the risk free rate is 6% and market risk premium is 10%, what is the expected return for the overall stock market? What is the required rate of return on a stock that has beta of 1.6?

**Solution:**

We know, Risk Premium = ($R_m$-$R_f$), here Risk Premium = 10%

So, $R_m = R_f + 10 = 6 + 10 = 16$

Required Rate of Return, $E(R_i) = R_f + (R_m - R_f)\beta$

$$E(R_i) = 6 + (10)1.6 = 22\%$$

**2-9: (NU BBA – 2013)** Following are the probability distribution and return of two securities:-

| Scenario | PROBABILITY | RATES OF RETURN | |
|---|---|---|---|
| | | A | B |
| Recession | 0.25 | -.08 | .20 |
| Normal | 0.50 | .03 | .05 |
| Boom | 0.25 | .20 | -.25 |

1. If weight of security A is 75% and B is 25% find out the portfolio return and portfolio standard deviation.

COVAB

# 2. If covariance between A and B is 70% then what is the standard deviation of the portfolio? (In the box)

**Solution:**

I. Portfolio Return, $\bar{x}_P = \sum_{i=1}^{n} w_i \bar{x}_i. = (.75 \times .0405) + (.25 \times 0.005) = .031625$

***Calculation of expected rate of return:***

$\bar{x}_A = (.25 \times -.08) + (.5 \times .03) + (.25 \times .20)$ = 0.0405 ⇒ .045

$\bar{x}_B = (.25 \times .20) + (.5 \times .05) + (.25 \times -.25)$ = 0.005 ⇒ 0.125

Portfolio standard deviation, $\sigma_P = \sqrt{w_A^{\ 2} \sigma_A^{\ 2} + w_B^{\ 2} \sigma_B^{\ 2} + 2 w_A w_B COV_{AB}}$

***Calculation of standard deviation:***

$$\sigma_A = \sqrt{.25(-.08 - .0405)^2 + .5(.03 - 0.0405)^2 + .25(.20 - .0405)^2}$$
$$= \sqrt{.01004525} = .100226$$

$$\sigma_B = \sqrt{.25(.20 - .005)^2 + .5(.05 - .005)^2 + .25(-.25 - .005)^2}$$
$$= \sqrt{.026772} = .163631$$

$$\sigma_p = \sqrt{(.75)^2 (.100226)^2 + (.25)^2 (.162699877)^2 + (2 \times .75 \times .25 \times -.01627875)}$$
$$= \sqrt{.0521} = .2282$$

Covariance, $COV_{AB} = \sum_{i=1}^{n} p_{AB} (x_A - \bar{x}_A)(x_B - \bar{x}_B).$

$$COV_{AB} = .25(-.08 - .0405)(.20 - .005) + .5(.03 - .0405)(.05 - .005)$$
$$+ .25(.20 - .0405)(-.25 - .005) = -.01627875$$

II. $\sigma_p = \sqrt{(.75)^2 (.100226)^2 + (.25)^2 (.162699877)^2 + (2 \times .75 \times .25 \times .70)}$

$$= \sqrt{.26980} = .51945$$

**2-10: (NU BBA – 2015 )** At present, suppose the risk free rate is 11% and the expected return on the market portfolio is 15%. The expected returns for four stocks are listed together with their expected beta:-

| Stock | Expected Return | Expected Beta |
|---|---|---|
| A | 17% | 1.35 |
| B | 16% | 0.85 |
| C | 15% | 1.20 |
| D | 18% | 1.75 |

On the basis of the expectations, which stocks are overvalued and undervalued?

**Solution:** We know, Expected rate of return, $E(R_i) = R_f + (R_m - R_f)\beta$

For A, $E(R_A) = .11 + (.15 - .11)1.35 = 0.1640 = 16.40\%$

For B, $E(R_A) = .11 + (.15 - .11)\ .85 = 0.1440 = 14.40\%$

For C, $E(R_A) = .11 + (.15 - .11)\ 1.20 = 0.1580 = 15.80\%$

For D, $E(R_A) = .11 + (.15 - .11)\ 1.75 = 0.18 = 18\%$

* actual এর চেয়ে Expected বেশি হলে overvalued.
* actual এর চেয়ে Expected কম হলে Undervalued.

| Stock | Expected Return | Actual Return | Market Status |
|---|---|---|---|
| A | 17 | 16.4 | Over Valued |
| B | 16 | 14.4 | Over Valued |
| C | 15 | 15.8 | Under Valued |
| D | 18 | 18 | Equilibrium |

**2-11: (NU BBA – 2015 )** Suppose that three investment projects, A, B and C have the following estimated rate of returns are shown as follows :-

| State of the Economy | Probability of Each State Occurring | Rates of Return if State Occurs | | |
|---|---|---|---|---|
| | | A | B | C |
| **Mild Boom** | 0.30 | -15% | 10% | 14% |
| **Boom** | 0.40 | 14 | 13 | -12 |
| **Strong Boom** | 0.30 | 16 | -20 | 10 |

Requirements:

i. Expected return for each projects., ii. Standard deviation of each project.

i.ii. Coefficient of variation of all projects., iv. Rank the alternatives on the basis of (1) expected return (2) risk. Which alternative would you choose?

**Solution:**

**a. Calculation of expected rate of return:**

$$\bar{x}_A = (-.15 \times .30) + (.14 \times .40) + (.16 \times .30) = 0.059$$

$$\bar{x}_B = (.10 \times .30) + (.13 \times .40) + (-.20 \times .30) = 0.022$$

$$\bar{x}_c = (.14\times.30)+(-.12\times.40)+(.10\times.30) = 0.024$$

**Calculation of standard deviation:**

$$\sigma_A = \sqrt{.30(-.15-0.059)^2+.40(.14-.059)^2+.3(.16-.059)^2}$$
$$= \sqrt{0.018796}$$
$$= 0.1371$$

$$\sigma_B = \sqrt{.30(.14-0.022)^2+.40(.13-.022)^2+.3(-.20-.022)^2}$$
$$= \sqrt{0.021276}$$
$$= 0.1459$$

$$\sigma_C = \sqrt{.30(.14-.024)^2+.40(-.12-.024)^2+.30(.10-.024)^2}$$
$$= \sqrt{0.014064}$$
$$= 0.1186$$

**Calculation of Coefficient of Variation:**

$CV_A = \frac{\sigma_A}{\bar{R}_A}$

$$CV_A = \frac{0.1371}{0.059} = 2.324\%$$

$$CV_B = \frac{0.1459}{0.022} = 6.63\%$$

$$CV_C = \frac{0.1186}{0.024} = 4.94\%$$

**b. Rank according to expected rate of return:**

$\bar{x}_A = 0.059$ Higher return

$\bar{x}_c = 0.024$ Moderate return

$\bar{x}_B = 0.022$ Lower Return

**Rank according to risk:**

$CV_B = 6.63\%$ Highly risky

$CV_C = 4.94\%$ Moderately risky

$CV_A = 2.324\%$ Less risky

I will choose alternative A because it gives same level of return at a lower risk level (findings of CV)

## SOLVED PROBLEMS

**2-1** From the following information of two investment projects each costing $40,000, you are required to calculate the Expected Return of the projects:

| Project X | | Project Y | |
|---|---|---|---|
| Probability | Cash flow ($) | Probability | Cash flow ($) |
| 0.10 | 13,000 | 0.10 | 12,000 |
| 0.20 | 13,500 | 0.25 | 13,000 |
| 0.40 | 14,000 | 0.30 | 14,000 |
| 0.20 | 14,500 | 0.25 | 15,000 |
| 0.10 | 15,000 | 0.10 | 16,000 |

**Solution:**

We can calculate riskiness of the cash flows of the project through calculating Expected rate of Return of each project:

We know, Expected rate of Return, $\bar{x}_i = \sum_{i=1}^{n} x_i P_i$

For project- X:

Expected rate of Return,

$$\bar{x}_X = (13000\times.1)+(13500\times.2)+(14000\times.4)+(14500\times.2)+(15000\times.1)$$

= 14000

For project- Y:

Expected rate of Return,

$$\bar{x}_Y = (12000\times.1)+(13000\times.25)+(14000\times.3)+(15000\times.25)+(16000\times.1)$$

= 14000

**2-2** Stock A and stock B have the following probability distribution of expected future returns:

| Probability | A | B |
|---|---|---|
| 0.1 | -0.25 | -0.40 |
| 0.2 | 0.05 | 0.00 |
| 0.4 | 0.15 | 0.06 |
| 0.2 | 0.30 | 0.40 |
| 0.1 | 0.45 | 0.66 |

a. Calculate the expected rate of return for stock A and stock B.

b. Calculate the standard deviation and coefficient of variation of expected returns for stock A and stock B. Is it possible that most investors might regard stock B as being less risky than stock A? Explain.

**Solution:**

a. We know, Expected rate of Return, $\bar{x}_i = \sum_{i=1}^{n} x_i P_i$

$$\bar{x}_A = (-.25\times.1) + (.05\times.2) + (.15\times.4) + (.30\times.2) + (.45\times.1)$$
$$= 0.15$$

$$\bar{x}_B = (-.40\times.1) + (.00\times.2) + (.06\times.4) + (.40\times.2) + (.66\times.1)$$
$$= 0.13$$

b. Standard deviation, $\sigma_i = \sqrt{\sum_{i=1}^{n} P_i(x_i - \bar{x})^2}$

$$\sigma_A = \sqrt{.1(-.25-.15)^2 + .2(.05-.15)^2 + .4(.15-.15)^2 + .2(.3-.15)^2 + .1(.45-.15)^2}$$
$$= \sqrt{0.0315}$$
$$= 0.1775$$

$$\sigma_B = \sqrt{.1(-.40-.13)^2 + .2(.00-.13)^2 + .4(.06-.13)^2 + .2(.4-.13)^2 + .1(.66-.13)^2}$$
$$= \sqrt{0.0761}$$
$$= 0.2759$$

**We know, Coefficient of Variation, CV=** $\frac{\sigma_i}{\bar{x}_i}$

$$CV_A = \frac{0.1775}{0.15} = 1.18$$

$$CV_B = \frac{0.2759}{.13} = 2.12$$

***Explanation:*** No, it's not possible. Most investors can not regard stock- B as being less risky. Since, stock B's CV (2.12) is higher than the stock A's CV (1.18), stock B is more risky than stock Y.

**2-3** The Berry Corporations is considering three possible capital projects for next year. Each project has a 1-year life, and project returns depend on next year's state of the economy. The estimated rates of return are shown in the table:

| State of the Economy | Probability of Each State Occurring | Rates of Return if State Occurs | | |
|---|---|---|---|---|
| | | **A** | **B** | **C** |
| **Recession** | 0.25 | 0.10 | 0.09 | 0.14 |
| **Average** | 0.50 | 0.14 | 0.13 | 0.12 |
| **Boom** | 0.25 | 0.16 | 0.18 | 0.10 |

a. Find each project's expected rate of return, variance, standard deviation, and coefficient of variation.

b. Rank the alternatives on the basis of (1) expected return and (2) risk. Which alternative would you choose?

**Solution:**

**a. Calculation of expected rate of return:**

$$\bar{x}_A = (.25\times.10)+(.50\times.14)+(.25\times.16) = 0.135$$

$$\bar{x}_B = (.25\times.09)+(.50\times.13)+(.25\times.18) = 0.1325$$

$$\bar{x}_c = (.25\times.14)+(.50\times.12)+(.25\times.10) = 0.12$$

**Calculation of standard deviation:**

$$\sigma_A = \sqrt{.25(.10-.135)^2+.50(.14-.135)^2+.25(.16-.135)^2}$$

$$= \sqrt{0.000475}$$

$$= 0.0218$$

$$\sigma_B = \sqrt{.25(.09-.1325)^2+.50(.13-.1325)^2+.25(.18-.1325)^2}$$

$$= \sqrt{0.00101875}$$

$= 0.032$

$$\sigma_C = \sqrt{.25(.14-.12)^2 + .50(.12-.12)^2 + .25(.10-.12)^2}$$

$$= \sqrt{0.0002}$$

$$= 0.01414$$

**Calculation of Variance:**

$$\sigma_A{}^2 = (0.0218)^2 = 0.000475$$

$$\sigma_B{}^2 = (0.032)^2 = 0.00102$$

$$\sigma_C{}^2 = (0.01414)^2 = 0.0002$$

**Calculation of Coefficient of Variation:**

$$CV_A = \frac{0.0218}{0.135} = 0.1615 = 16.15\%$$

$$CV_B = \frac{0.032}{0.1325} = 0.2415 = 24.15\%$$

$$CV_C = \frac{0.01414}{0.12} = 0.1178 = 11.78\%$$

**b. Rank according to expected rate of return:**

$\bar{x}_A = 0.135$ Higher return

$\bar{x}_B = 0.1325$ Moderate return

$\bar{x}c = 0.12$ Lower Return

**Rank according to risk:**

$CV_B = 24.15\%$ Highly risky

$CV_A = 16.15\%$ Moderately risky

$CV_C = 11.78\%$ Less risky

I will choose alternative C because it gives same level of return at a lower risk level (findings of CV)

Scanned with CamScanner

**2-4** Refer to the three alternative projects contained in exercise 3. Assume that the Berry Corporation is going to invest one-third of its available funds in each project. That is, Berry will create a portfolio of three equally weighted projects.

a. What is the expected rate of return on the portfolio?

b. What are the variance and standard deviation of the portfolio?

c. What are the covariance and correlation coefficient between projects A and B? Between projects A and C?

**Solution:**

a. We know, Portfolio return, $\bar{x}_P = \sum_{i=1}^{n} w_i \bar{x}_i.$

$$= (.333 \times .135) + (.333 \times 0.1325) + (.333 \times .12)$$

$$= 0.129$$

b. We know, Portfolio standard deviation,

$$\sigma_P = \sqrt{w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + 2w_A w_B COV_{AB} + 2w_B w_C COV_{BC} + 2w_C w_A COV_{CA}}$$

$$= \sqrt{\begin{matrix}(.333)^2(.0218)^2 + (.333)^2(.032)^2 + (.333)^2(.01414)^2 + (2 \times .333 \times 333 \times .00066) \\ + (-0.00045 \times 2 \times .333 \times .333) + (-0.0003 \times 2 \times .333 \times .333)\end{matrix}}$$

$$= \sqrt{0.00016846} = 0.013$$

Variance, $\sigma_P^2 = (0.013)^2 = 0.00017$

c. We know, Covariance, $COV_{AB} = \sum_{i=1}^{n} p_i (x_A - \bar{x}_A)(x_B - \bar{x}_B)$

$$COV_{AB} = .25(.10 - .135)(.09 - .1325) + .5(.14 - .135)(.13 - .1325) + .25(.16 - .135)(.18 - .1325)$$

$$= 0.00066$$

$$COV_{CA} = .25(.10 - .135)(.14 - .12) + .5(.14 - .135)(.12 - .12) + .25(.16 - .135)(.10 - .12)$$

$$= -0.0003$$

$$COV_{BC} = .25(.09 - .1325)(.14 - .12) + .5(.13 - .1325)(.12 - .12) + .25(.18 - .1325)(.10 - .12)$$

$$= -0.00045$$

We know,

$$\text{Correlation, } \rho_{AB} = \frac{COV_{AB}}{\sigma_A \sigma_B} = \frac{0.00066}{.0218 \times .032} = 0.9461$$

$$\text{Correlation, } \rho_{AC} = \frac{COV_{AC}}{\sigma_A \sigma_C} = \frac{-0.0003}{.0218 \times .01414} = -0.9732$$

**2-5** Stock A and B have the following historical returns:

| Year | Stock A's Returns | Stock B's Returns |
|---|---|---|
| 2000 | -0.18 | -0.24 |
| 2001 | 0.44 | 0.24 |
| 2002 | -0.22 | -0.04 |
| 2003 | 0.22 | 0.08 |
| 2004 | 0.34 | 0.56 |

a. Calculate the average rate of return for each stock during the 5-year period. Assume that someone held a portfolio consisting of 50 percent of stock A and 50 percent of stock B. What would have been the average return on the portfolio during this period?

b. Now calculate the standard deviation of re turns for each stock and for the portfolio.

c. Looking at the annual returns data on the two stocks, would you guess that the correlation coefficient between return on the two stocks is closer to 0.8 or -0.8?

**Solution:** (a) Expected return for stocks :

$$\bar{x}_A = \frac{-.18 + .44 - .22 + .22 + .34}{5} = 0.12$$

$$\bar{x}_B = \frac{-.24 + .24 - .04 + .08 + .56}{5} = 0.12$$

Portfolio return, $\bar{x}_P = (.50 \times .12) + (.50 \times .12) = 0.12$

(b) Standard deviation for stocks :

$$\sigma_A = \sqrt{\frac{(-.18 - .12)^2 + (.44 - .12)^2 + (-.22 - .12)^2 + (.22 - .12)^2 + (.34 - .12)^2}{5 - 1}}$$

$$= \sqrt{0.0916} = 0.30265$$

$$\sigma_B = \sqrt{\frac{(-.24-.12)^2+(.24-.12)^2+(-.04-.12)^2+(.08-.12)^2+(.56-.12)^2}{5-1}}$$

$$= \sqrt{0.0912} = 0.302$$

**Portfolio standard deviation,**

$$\sigma_P = \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B COV_{AB}}$$

$$= \sqrt{(.5)^2(.30265)^2 + (.5)^2(.302)^2 + (2\times.5\times.5\times0.0734)}$$

$$= 0.0824$$

Here,

$$\textbf{Covariance, } COV_{AB} = \frac{\sum_{i=1}^{n}(x_A - \bar{x}_A)(x_B - \bar{x}_B)}{n-1}$$

$$= \frac{(-.18-.12)(-.24-.12)+(.44-.12)(.24-.12)+(-.22-.12)(-.04-.12)+(.22-.12)(.08-.12)+(.34-.12)(.56-.12)}{5-1}$$

$$= 0.0734$$

(c) We know, Correlation coefficient, $\rho_{AB} = \frac{COV_{AB}}{\sigma_A \sigma_B} = \frac{0.0734}{0.30265\times0.302} = 0.803$

Yes, correlation coefficient of two stocks is closer to 0.8.

**2-6** Based on 5 years of monthly data, you derive the following information for the companies listed:

| Company | $\sigma_i$ | $\rho_{im}$ |
|---|---|---|
| Intel | 0.1210 | 0.72 |
| Ford | 0.1460 | 0.33 |
| Anheuser Busch | 0.0760 | 0.55 |
| Merck | 0.1020 | 0.60 |
| S&P 500 | 0.0550 | 1.00 |

a. Calculate the beta coefficient for each stock.

b. Assuming a risk-free rate of 8 percent and an expected return for the market portfolio is 15 percent, compute the expected (required) return for all the stocks and plot them on the SML.

**Solution: We know,** $\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$

a. $\beta_{Intel} = \frac{\sigma_i \sigma_m \rho_{im}}{\sigma_m^2} = \frac{.1210 \times .0550 \times .72}{(.0550)^2} = 1.584$

$$\beta_{Ford} = \frac{.1460 \times .0550 \times .33}{(.0550)^2} = .876$$

$$\beta_{AnheuserBush} = \frac{.0760 \times .0550 \times .55}{(.0550)^2} = .76$$

$$\beta_{Merck} = \frac{.1020 \times .0550 \times .60}{(.0550)^2} = 1.113$$

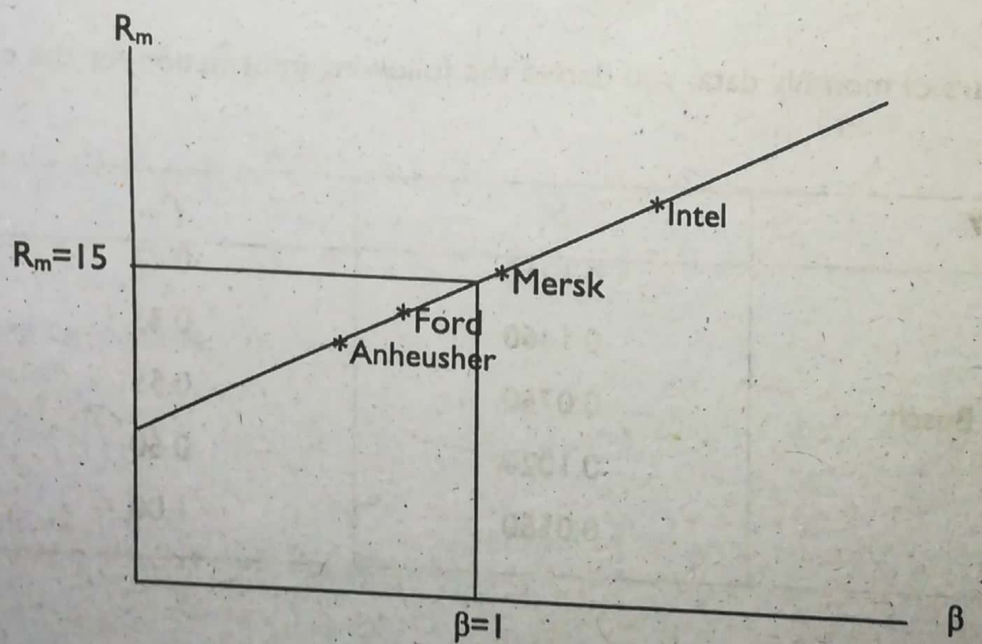
Required Rate of Return, $E(R_i) = R_f + (R_m - R_f)\beta$

$$E(R_{Intel}) = .08 + (.15 - .08)1.584 = .1909 \, or \, 19.09\%$$

$$E(R_{Ford}) = .08 + (.15 - .08).876 = .1413$$

$$E(R_{AnheusherBush}) = .08 + (.15 - .08).76 = .1332$$

$$E(R_{Merck}) = .08 + (.15 - .08)1.113 = .1573$$



2-7 Calculate the expected (required) return for each of the following stocks when the risk-free rate is 0.08 and you expect the market return to be 0.14.

| Stock | Beta |
|---|---|
| A | 1.72 |
| B | 1.14 |
| C | 0.76 |
| D | 0.44 |
| E | 0.03 |
| F | -0.79 |

**Solution:** We know, Expected rate of return, $E(R_i) = R_f + (R_m - R_f)\beta$

For A, $E(R_A) = .08 + (.14 - .08)1.72 = 0.1832 = 18.32\%$

For B, $E(R_B) = .08 + (.14 - .08)1.14 = 0.1484 = 14.84\%$

For C, $E(R_C) = .08 + (.14 - .08)0.76 = 0.1256 = 12.56\%$

For D, $E(R_D) = .08 + (.14 - .08)0.44 = 0.1064 = 10.64\%$

For E, $E(R_E) = .08 + (.14 - .08)0.03 = 0.0818 = 8.18\%$

For F, $E(R_F) = .08 + (.14 - .08)(-0.79) = 0.0326 = 3.26\%$

**2-8** The following are the historical returns for the Chelle Computer Company:

| Year | Chelle Computer | General Index |
|---|---|---|
| 1 | 0.37 | 0.15 |
| 2 | 0.09 | 0.13 |
| 3 | -0.11 | 0.14 |
| 4 | 0.08 | -0.09 |
| 5 | 0.11 | 0.12 |
| 6 | 0.04 | 0.09 |

Based on this information, compute the following:

a. The correlation coefficient between Chelle Computer and the General Index.
b. The standard deviation for the company and the index.
c. The beta for the Chelle Computer Company.

**Solution:**

a. We know,

$$\text{Correlation, } \rho_{CI} = \frac{COV_{CI}}{\sigma_C \sigma_I} = \frac{0.00184}{0.1556 \times 0.09} = 0.1314$$

Here, Expected return:

$$\bar{x}_C = \frac{.37 + .09 - .11 + .08 + .11 + .04}{6} = 0.097$$

$$\bar{x}_I = \frac{.15 + .13 + .14 - .09 + .12 + .09}{6} = 0.09$$

***Covariance,***

$$COV_{CI} = \frac{(.37 - .097)(.15 - .09) + (.09 - .097)(.19 - .09) + (-.11 - .097)(.14 - .09) + (.08 - .097)(-.09 - .09) + (.11 - .097)(.12 - .09) + (.04 - .097)(.09 - .09)}{6 - 1}$$

$$= \frac{0.0092}{5} = 0.00184$$

***b. We know, Standard deviation,*** $\sigma_i = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x}_i)^2}{n-1}}$

$$\sigma_C = \sqrt{\frac{(.37 - .097)^2 + (.09 - .097)^2 + (-.11 - .097)^2 + (.08 - .097)^2 + (.11 - .097)^2 + (.04 - .097)^2}{6 - 1}}$$

$$= \sqrt{0.02423} = 0.1556$$

$$\sigma_I = \sqrt{\frac{(.15 - .09)^2 + (.13 - .09)^2 + (.14 - .09)^2 + (-.09 - .09)^2 + (.12 - .09)^2 + (.09 - .09)^2}{6 - 1}}$$

$$= \sqrt{0.0082} = 0.09$$

***c. We know, Beta coefficient,*** $\beta_C = \frac{COV_{CI}}{\sigma_I^2}$

$$= \frac{0.00184}{(0.09)^2} = 0,227$$

**2-9** ECRI Corporation is a holding company with four main subsidiaries. The percentage of its business coming from each of the subsidiaries, and their respective betas, are as follows:

| Subsidiary | Percentage of Business | Beta |
|---|---|---|
| Electric utility | 60% | 0.70 |
| Cable company | 25 | 0.90 |
| Real estate | 10 | 1.30 |
| International projects | 5 | 1.50 |

a. What is the holding company's beta?

b. Assume that the risk-free rate is 6 percent and the market risk premium is 5 percent. What is the holding company's required rate of return?

**Solution:**

a. We know,

Company beta, $\beta_C = \sum_{i=1}^{n} w_i \beta_i$.

$$= (.70\times.6)+(.25\times.9)+(.1\times1.30)+(.05\times1.50)$$

$$= 0.67$$

b. Holding company's return, $E(R_C) = .06 + (.11 - .06)0.67 = 0.0935$

**2-10** Suppose $R_f$ = 9%, $R_m$ = 14%, and $\beta_i$ = 1.3.

a. What is $R_i$, the required rate of return on stock i?

b. Now suppose $R_f$ (1) increases to 10 percent or (2) decreases to 8 percent. The slope of the SML remains constant. How would this affect $R_m$ and $R_i$?

c. Now assume $R_f$ remains at 9 percent but $R_m$ (1) increases to 16 percent or (2) falls to 13 percent. The slope of the SML does not remain constant. How would these changes affect $R_i$?

**Solution:**

a. We know, Required rate of return, $E(R_i) = R_f + (R_m - R_f)\beta$

$$= 0.09+(.14-.09)1.3$$

$$= 0.155 \text{ or } 15.5\%$$

b. i. When $R_f$ increases to 10%, the slope of the SML $(R_m - R_f)$ remains same so the new $R_m$ will be 15% due to the increase in $R_f$.

Required rate of return, $E(R_i) = R_f + (R_m - R_f)\beta$ = 0.10+(.15-.10)1.3 = 0.165 or 16.5%

ii. When $R_f$ decreases to 8%, the slope of the SML $(R_m - R_f)$ remains same so the new $R_m$ will be 13% due to the increase in $R_f$.

Required rate of return, $E(R_i) = R_f + (R_m - R_f)\beta$ = 0.08+(.13-.08)1.3 = 0.145 or 14.5%

c. i. When $R_m$ increases to 16%,

Required rate of return, $E(R_i) = R_f + (R_m - R_f)\beta$ = 0.09+(.16-.09)1.3 = 0.181 or 18.1%

ii. When $R_m$ falls to 13%,

Expected rate of return, $E(R_i) = R_f + (R_m - R_f)\beta$ = 0.09+(.13-.09)1.3 =0.142 or 14.2%

**2-11** You have a £2 million portfolio consisting of a £100,000 investment in each of 20 different stocks. The portfolio has a beta of 1.1. You are considering selling £100,000 worth of one stock which has a beta equal to 0.9 and using the proceeds to purchase another stock which has a beta equal to 1.4. What will be the new beta of your portfolio following this transaction?

**Solution:**

New b =old beta + (weight x change in beta)

=1.1 +.05 (1.4-0.9)

= 1.1 +.05 (.5) = 1.125

Or New b = Old beta – (weight of replaced security x beta of the replaced security) + (weight of replacing security x beta of the replacing security)

= 1.1 – (.05x.9) + (.05x1.4)

= 1.1 - .045 + .07 = 1.125

**2-12** Stock R has a beta of 1.5, stock S has a beta of 0.75, the expected rate of return on an average stock is 13 percent, and the risk-free rate of return is 7 percent. By how much does the required return on the riskier stock exceed the required return on the less risky stock?

**Solution:**

We know, required rate of return, $E(R_i) = R_f + (R_m - R_f)\beta$

For stock-R, $E(R_R) = 0.07 + (0.13 - 0.07)1.5 = 0.16 = 16\%$

For stock-S, $E(R_S) = 0.07 + (0.13 - 0.07)0.75 = 0.115 = 11.5\%$

Here Stock-R is more risky stock. Thus expected rate of return of riskier project stock-R exceed (16%-11.5%) 4.5% the rate of return on the less risky stock-S.

**2-13** Stock A and Stock B have the following historical returns:

| Year | Stock A's Returns | Stock B's Returns |
|---|---|---|
| 2010 | (18.00%) | (14.50%) |
| 2011 | 33.00 | 21.80 |
| 2012 | 15.00 | 30.50 |
| 2013 | (0.50) | (7.60) |
| 2014 | 27.00 | 26.30 |

a. Calculate the average rate of return for each stock during the 5-year period.

b. Assume that someone held a portfolio consisting of 50 percent of stock A and 50 percent of stock B. What would have been the realized rate of return on the portfolio in each year? What would have been the average return on the portfolio during this period?

c. Calculate the standard deviation of returns for each stock and for the portfolio.

d. Calculate the coefficient of variation for each stock and for the portfolio.

e. If you are a risk-averse investor, would you prefer to hold stock A, stock B, or the portfolio? Why?

**Solution:**

a. We know, Expected rate of return, $\bar{x}_i = \frac{\sum_{i=1}^{n} x_i}{n}$

$$\bar{x}_A = \frac{-18 + 33 + 15 - .50 + 27}{5} = 11.3\%$$

$$\bar{x}_B = \frac{-14.5+21.8+30.5-7.6+26.30}{5} = 11.3\%$$

b. Calculation for realized rate of each year:

| Year | | Portfolio Returns |
|---|---|---|
| 2010 | $(-18\times.5)+(-14.5\times.5)$ | = (16.25%) |
| 2011 | $(33\times.5)+(21.8\times.5)$ | = 27.4% |
| 2022 | $(15\times.5)+(30.5\times.5)$ | = 22.75% |
| 2013 | $(-.5\times.5)+(-7.6\times.5)$ | = (4.05%) |
| 2014 | $(27\times.5)+(26.3\times.5)$ | = 26.65% |

Portfolio average return, $\bar{x}_P = \frac{-16.25+27.4+22.75-4.05+26.65}{5} = 11.3\%$

c. We know, Standard deviation, $\sigma_i = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x}_i)^2}{n-1}}$

$$\sigma_A = \sqrt{\frac{(-18-11.3)^2+(33-11.3)^2+(15-11.3)^2+(-.5-11.3)^2+(27-11.3)^2}{5-1}}$$

$$= \sqrt{432.2} = 20.79\%$$

$$\sigma_B = \sqrt{\frac{(-14.5-11.3)^2+(21.8-11.3)^2+(30.5-11.3)^2+(-7.6-11.3)^2+(26.3-11.3)^2}{5-1}}$$

$$= \sqrt{431.685} = 20.78\%$$

$$\sigma_P = \sqrt{\frac{(-16.25-11.3)^2+(27.4-11.3)^2+(22.75-11.3)^2+(-4.05-11.3)^2+(26.65-11.3)^2}{5-1}}$$

$$= \sqrt{405.14} = 20.13\%$$

d. We know, Coefficient of variation, CV= $\frac{\sigma_i}{\bar{x}_i}$

$$CV_A = \frac{20.79}{11.3} = 1.84\%$$

$$CV_B = \frac{20.78}{11.3} = 1.84\%$$

$$CV_P = \frac{20.13}{11.3} = 1.78\%$$

e. If I am risk-averse investor, I prefer to invest in the portfolio because it provides the same return (11.3%) at lower level of risk.

**2-14** You have observed the following returns over time:

| Year | Stock X | Stock Y | Market |
|---|---|---|---|
| 2000 | 14% | 13% | 12% |
| 2001 | 19 | 7 | 10 |
| 2002 | -16 | -5 | -12 |
| 2003 | 3 | 1 | 1 |
| 2004 | 20 | 11 | 15 |

Assume that the risk-free rate is 6 percent and the market risk premium is 5 percent.

a. What are the betas of stocks X and Y?

b. What are the expected rates of return for stocks X and Y?

c. What is the expected rate of return for a portfolio consisting of 80 percent of stock X and 20 percent of stock Y?

d. If stock X's required return is 22 percent, is stock X under- or overvalued?

**Solution:**

Expected return for stocks:

$$\overline{R}_X = \frac{.14+.19-.16+.03+.20}{5} = 0.08$$

$$\overline{R}_Y = \frac{.13+.07-.05+.01+.11}{5} = 0.054$$

$$\overline{R}_M = \frac{.12+.10-.12+.01+.15}{5} = 0.052$$

Portfolio return, $\overline{R}_P = (.80\times.08)+(.20\times.054) = 0.0748$

$$COV_{XM} = \frac{\sum(X_i - \overline{X}_i)(X_M - \overline{X}_M)}{n-1}$$

$$Cov_{XM} = \frac{(.14-.08)(.12-.052)+(.19-.08)(.10-.052)+(-.16-.08)(-.12-.052)+(.03-.08)(.01-.052)+(.20-.08)(.15-.052)}{5-1}$$

$$= \frac{.0645}{5-1} = .016125$$

$$Cov_{YM} = \frac{(.13-.054)(.12-.052)+(.07-.054)(.10-.052)+(-.05-.054)(-.12-.052)+(.01-.054)(.01-.052)+(.11-.054)(.15-.052)}{5-1}$$

$$= \frac{.03116}{5-1} = .00779$$

Variance of Market,

$$\sigma_M^2 = \frac{\sum_{i=1}^{n}(x_M - \bar{x}_M)^2}{n-1}$$

$$= \frac{(.12-.052)^2 + (.10-.052)^2 + (-.12-.052)^2 + (.01-.052)^2 + (.15-.052)^2}{5-1}$$

$$= \frac{.04788}{4} = .01197$$

Beta coefficient, $\beta_X = \frac{COV_{XM}}{\sigma_M{}^2} = \frac{.016125}{.01197} = 1.34711779$

Beta coefficient, $\beta_X = \frac{COV_{YM}}{\sigma_M{}^2} = \frac{.00779}{.01197} = 0.65079$

| **Summary of Results** | | |
|---|---|---|
| **Particulars** | **Stock X** | **Stock Y** |
| Beta | 1.34711779 | .65079 |
| Expected Return | .08 | .054 |
| Portfolio return | .0748 | |
| As the Expected Return is less than Required Return Stock X is said to be Overvalued | | |

**2-15** You have a portfolio of the following four shares:

| Share | Beta | Investment (Tk.) |
|---|---|---|
| A | 0.90 | 2,00,000 |
| B | 1.20 | 2,20,000 |
| C | 1.00 | 1,80,000 |
| D | 0.80 | 2,00,000 |

What is the expected rate of return on your portfolio if the risk free rate of return is 9 percent and the expected market rate of return is 16 percent?

**Solution:**

| Share | $\beta_i$ | Investment | $w_i$ | $w_i\beta_i$ |
|---|---|---|---|---|
| A | 0.90 | 200,000 | 0.250 | 0.225 |
| B | 1.20 | 220,000 | 0.275 | 0.330 |
| C | 1.00 | 180,000 | 0.225 | 0.225 |
| D | 0.80 | 200,000 | 0.250 | 0.200 |

$\sum w_i\beta = 0.98_i$

Portfolio Beta, $\sum w_i\beta = 0.98_i$

Risk Free Rate of Return = $R_f$ = 9% = .09

Market rate of Return = $R_m$ = 16% = .16

Portfolio return= $R_p = R_f + \beta i\,(R_m - R_f) = .09 + 0.98(.16 - .09) = 15.86$

**2-16** The historical required rate of return of market portfolio and security R are given below. Considering the following information you are required to calculate the systematic risk (beta factor) of security R.

| Year | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Security R | 2% | 3% | 6% | -4% | 8% |
| Market Portfolio | 4% | -2% | 8% | -4% | 4% |

***Solution***

| $R_i$ | $R_m$ | $(R_i - \overline{R}_i)$ | $(R_m - \overline{R}_m)$ | $(R_i - \overline{R}_i)(R_m - \overline{R}_m)$ | $(R_m - \overline{R}_m)^2$ |
|---|---|---|---|---|---|
| 0.02 | 0.04 | -0.01 | 0.02 | 0.0002 | 0.0004 |
| 0.03 | -0.02 | 0 | -0.04 | 0 | 0.0016 |
| 0.06 | 0.08 | 0.03 | 0.06 | 0.0018 | 0.0036 |
| -0.04 | -0.04 | -0.07 | -0.06 | 0.0042 | 0.0036 |
| 0.08 | 0.04 | 0,05 | -0.02 | 0.001 | 0.0004 |
| ***Σ=0.15*** | ***Σ=0.10*** | | | ***Σ=0.0068*** | ***Σ=0.0096*** |

Expected rate of return, $\overline{R}_i = \frac{\sum_{i=1}^{n} R_i}{n}$

$$\overline{R}_A = \frac{.15}{5} = .03$$

$$\overline{R}_m = \frac{.10}{5} = .02$$

c. We know,

$$\sigma^2{}_m = \frac{\sum_{i=1}^{n}(R_m - \overline{R}_m)^2}{n-1} = \frac{.0096}{5-1} = 0.0024$$

$$\sigma_{im} = \frac{\sum_{i=1}^{n}(R_i - \overline{R}_i)(R_m - \overline{R}_m)}{n-1} = \frac{.0068}{5-1} = 0.0024$$

$$\beta_R = \frac{\sigma_{im}}{\sigma_m^2} = \frac{0.0017}{.0024} = 0.70833333$$

**2-17** From the following information of two-investment project each costing Tk. 40000, you are required to calculate the Expected Return, Standard deviation and Co- efficient of variation of the project.

| PROJECT- X | | PROJECT-Y | |
|---|---|---|---|
| PROBABILITY DISTRIBUTION | CASH FLOW (TK.) | PROBABILITY DISTRIBUTION | CASH FLOW (TK.) |
| .10 | 13000 | .10 | 12000 |
| .20 | 13500 | .25 | 13000 |
| .40 | 14000 | .30 | 14000 |
| .20 | 14500 | .25 | 15000 |
| .10 | 15000 | .10 | 16000 |

**Solution:**

a. calculation of expected return of two-investment project:

$$\overline{X}_x = \sum \Pr i.xi$$

= (0.10X13000) + (0.20X13500) + (0.40X14000) + (0.20X14500) + (0.10X15000)

= 1300+2700+5600+2900+1500

= 14000

$$\overline{X}_y = \sum \Pr i.Xi$$

= (0.10X12000) + (0.25X13000) + (0.30X14000) + (0.25X15000) + (0.10X16000)

= 1200+3200+4200+3750+1600

= 14000

b. Calculation of standard deviation of two-invested project:

$$\sigma_x = \sqrt{\sum P_i (Xi - \overline{X})^2}$$

$$= \sqrt{\begin{array}{l}0.10(13000-14000) + 0.20(13500-14000) + 0.40(14000-14000) + \\ 0.20(14500-14000) + 0.10(15000-14000)\end{array}}$$

=5.47.72

$$\sigma_y = \sqrt{\sum P_i(Yi-\overline{Y})^2}$$

$$= \sqrt{\begin{matrix}0.10(12000-14000) + 0.25(13000-14000) + 0.30(14000-14000) + \\ 0.25(15000-14000) + 0.10(16000-14000)\end{matrix}}$$

$=1140.18$

c. Calculation of co-efficient of variation of two-investment projects

$$C.V(x) = \frac{\sigma_X}{\overline{X}_X} \times 100 = \frac{547.72}{14000} \times 100 = 3.91\%$$

$$C.V(y) = \frac{\sigma_Y}{\overline{X}_Y} \times 100 = \frac{1140.18}{14000} \times 100 = 8.14$$

**2-18:** From the following three economic condition calculate the Expected return and Standard Deviation of Assets - A and Assets - B.

| POSSIBLE OUTCOMES | PROBABILITY | RETURNS | |
|---|---|---|---|
| | | ASSET-A | ASSET-B |
| Pessimistic | .25 | 13% | 7% |
| Most likely | .50 | 15% | 15% |
| Optimistic | .25 | 17% | 25% |
| Total | 1.00 | | |

**Solution:**

a. Calculation of expected return of two Assets, A and B:

$$\overline{X}_A = \sum P_i x_i = (0.25X0.13) + (0.50 + 0.15) + (0.25X0.17) = .15 \, or \, 15\%$$

$$\overline{X}_B = (0.07X0.25) + (0.15X0.50) + (0.25X0.25) = 15.5\%$$

b. Calculation of standard deviation of two assets, A and B:

$$\sigma_A = \sqrt{\sum P_i(X_i - \overline{X}_A)^2} = \sqrt{0.25(013-0.15)^2 + 0.50(0.15-0.15)^2 + 0.25(0.17-0.155)^2}$$

$=1.41\%$

$$\sigma_B = \sqrt{\sum P_i(X_i - \overline{X}_B)^2} = \sqrt{(0.25(0.07-0.155)^2 + 0.50(0.15-0.155)^2 + 0.25(0.25-0.155)^2}$$

$=6.39\%$

**2-19:** Mr. Rhyme Sobhan a financial analyst for *GLAXO* wishes to estimate the rate of return for two similar risk investments, X and Y. Mr. Sobhan research indicates that the immediate past returns will serve as reasonable estimates of future returns. A year earlier investment X had a market value of TK. 20,000 investment in Y of Tk. 55,000. During the year, investment X generated cash flow of $ 1,500 and investment Y generated cash flow of Tk. 6,800. The current market values of investments X, and Y Tk. 21,000 and Tk.55,000 respectively.

a. Calculate the expected rate of return on investments X and Y using the most recent year's data.

b. Assuming that the two investments are equally risky, which one should Ahmed recommended? Why?

**Solution:**

a. Calculation o expected return on investments X and Y:

$$R_i = \frac{Cash\ flow + Ending\ value - Beginning\ value}{Beginning\ value}$$

Investments X:= $\frac{1500 + 21000 - 20000}{20000}$ = 12.5%

Investments Y: = $\frac{6800 + 55000 - 55000}{55000}$ = 0.1236, or 12.36%

b. Since, the two investments are equally risky; Ahmed should recommend investment X because the return of investment X is higher than investment Y.

**2-20:** For each of the investments shown in the following table calculate the rate of return earned over the specified time period.

| INVESTMENTS | CASH FLOW DURING | BEGINNING OF THE PERIOD | ENDING PERIOD VALUE |
|---|---|---|---|
| A | -$100 | $800 | $1,100 |
| B | 15,000 | 1,20,000 | 1,18,000 |
| C | 7000 | 45,000 | 48,000 |
| D | 80 | 600 | 500 |
| E | 1,500 | 12,500 | 12,400 |

**Solution:** Calculation of rate of return of five investments over the specified time period:

$$R_i = \frac{Cash\ flow + Ending\ value - Beginning\ value}{Beginning\ value}$$

**Investment A:** $= \dfrac{-100 + 1100 - 800}{800} = 0.25$

**Investment B:** $= \dfrac{15000 + 118000 - 120000}{120000} = 0.1083$

**Investment C:**$= \dfrac{7000 + 48000 - 45000}{45000} = 0.2222$

**Investment D:** $= \dfrac{80 + 500 - 600}{600} = -3.33\%$

**Investment E:** $= \dfrac{1500 + 12400 - 12500}{12500} = 11.2\%$

Now, Chronological order of investments over the specified time period: A, C, E, B, D

**2-21:** *Kaspersky Lab Inc.* is considering the purchase of one of two *Lab Computer*, B and S. Both should provide a benefit over a 10 year period, and each requires an initial investment of $ 4000. Management has constructed the following table of estimates of rates of return and probabilities for pessimistic, most likely and optimistic results:

| PARTICULARS | COMPUTER B | | COMPUTER S | |
|---|---|---|---|---|
| | AMOUNT | PROBABILITY | AMOUNT | PROBABILITY |
| Initial Investment | $ 4000 | 1.00 | $4000 | 1.00 |
| Annual rate of return | | | | |
| Pessimistic | 20% | .25 | 15% | .25 |
| Most Likely | 25% | .50 | 25% | .50 |
| Optimistic | 30% | .25 | 35% | .25 |

a. Determine the range for the rate of return for each of the two cameras.

b. Determine the expected value of return for each camera.

c. Purchase of each camera is riskier? Why?

**Solution:**

a. Calculation of the range for the return of two cameras:

Camera B= 30%-20% = 10%

Camera C= 35%-15% = 20%

b. Calculation of expected rate of return of two cameras:

$$\overline{X}_B = \sum P_i X_i = (0.25X0.20) + (.50X0.25) + (0.25X0.30)$$
$$= 0.05 + 0.12 + 0.075 = 0.25 = 25\%$$

$$\overline{X}_S = \sum \Pr i.Xi$$

= (0.25X0.15) + (0.50X0.25) + (0.25X0.35)

= 0.0375+0.125+0.0875 = .25 or 25%

A) Calculation of standard deviation of two cameras:

$$\sigma_B = \sqrt{\sum \Pr i(Xi - \overline{X}_B)^2}$$

$$= \sqrt{0.25(0.20 - 0.25)^2 + 0.50(0.25 - 0.255)^2 + 0.25(0.30 - 0.25)^2} = .0354\, or\, 3.54\%$$

$$\sigma_S = \sqrt{\sum \Pr i(Xi - \overline{X}_S)^2}$$

$$= \sqrt{0.25(0.15 - 0.25)^2 + 0.50(0.25 - 0.25)^2 + 0.25(0.35 - 0.25)^2} = .0707\, or\, 7.07\%$$

Now, Calculation of co-efficient of variation two cameras:

$$C.V(B) = \frac{\sigma_B}{\overline{X}_B} \times 100 = \frac{0.0354}{0.25} \times 100 = 14.16\%$$

$$C.V(S) = \frac{\sigma_S}{\overline{X}_S} \times 100 = \frac{.0707}{.25} \times 100 = 28.25\%$$

Camera S is more risky because it has higher standard deviation and higher co-efficient of variation than camera B.

**2-22:** Greengage Inc. a successful businessman is considering several expansion projects. All of the alternatives promise to produce an acceptable return. The owner is extremely risk averse; therefore, they will choose the least risky of the alternatives. Data on four possible projects follow:

| PROJECT | EXPECTED RETURN | RANGE | STANDARD DEVIATION |
|---|---|---|---|
| A | 12.0% | .040 | .029 |
| B | 12.5% | .050 | .032 |
| C | 13.0% | .060 | .035 |
| D | 12.8% | .045 | .030 |

a. Which project is least risky judging on the basis of range?

b. Which project has the lowest standard deviation? Explain why standard deviation is not an appropriate measure of risk for purposes of this comparison.

c. Calculate the coefficient of variation for each project. Which project will Greengage's owners choose?

**Solution:**

a) Project A is least risky judging on the basis of range because it has lower rate for range than Project B, C, D.

b) Project A has the lowest standard deviation. Standard deviation is not an appropriate measurement of risk for purpose of this comparison because standard deviation considers the absolute measurement rather than relative measurement.

c) Calculation of the co-efficient of variation for each projects:

$$C.V\ (A) = \frac{\sigma_A}{\overline{X}_A} \times 100 = \frac{0.029}{0.12} \times 100 = 24.17\%$$

$$C.V\ (B) = \frac{\sigma_B}{\overline{X}_B} \times 100 = \frac{0.032}{0.125} \times 100 = 25.6\%$$

$$C.V\ (C) = \frac{\sigma_C}{\overline{X}_C} \times 100 = \frac{0.035}{0.13} \times 100 = 26.92\%$$

$$C.V\ (D) = \frac{\sigma_D}{\overline{X}_D} \times 100 = \frac{0.03}{0.128} \times 100 = 23.44\%$$

Here, Greengages will choose Project D because it has lower co-efficient of variation

**2-23:** As the security analyst of Bay Finance and Investment you have been asked to give your advice in selecting a portfolio of securities and have been given the following data:

| Year | Expected Return | | |
|---|---|---|---|
| | BEXTEX | IFIC | G-2 |
| 2007 | 12% | 16% | 12% |
| 2008 | 14 | 14 | 14 |
| 2009 | 16 | 12 | 16 |

No probabilities have been supplied. You have been told that you can create two portfolios – one consisting of Bextex and IFIC and other consisting of assets Bextex and G-2– by investing equal proportions (50%) in each of the two component assets:

1. What is the expected return for each asset over the 3 year period?
2. What is the standard deviation for each assets return?
3. What is the expected return for each of the two portfolios?
4. How would you characterize the correlations of returns of the two assets making up each of the two portfolios identified in part 3?
5. What is the standard deviation for each portfolio?
6. Which portfolios do you recommended? Why?

**Solution: a.** Calculation of expected return for each asset over the three years:

$$E(r)_B = \frac{0.12+0.14+0.16}{3} = 0.14$$

$$E(r)_I = \frac{0.16+0.14+0.12}{3} = 0.14$$

$$E(r)_{G-2} = \frac{0.12+0.14+0.16}{3} = 0.14$$

a) Calculation of the standard deviation for each assets return:

$$\sigma_B = \sqrt{\frac{\sum(X-\bar{X}_A)^2}{n-1}}$$

$$= \sqrt{\frac{(0.12-0.14)^2+(0.14-0.14)^2+(0.16-0.14)^2}{3-1}}$$

$$= \sqrt{\frac{0.0008}{2}} = 0.02$$

Again,

$$\sigma_I = \sqrt{\frac{(0.16-0.14)^2+(0.14-0.14)^2+(0.12-0.14)^2}{3-1}}$$

$$= \sqrt{\frac{0.0004+0+0.0004}{2}} = 0.02$$

$$\sigma_G = \sqrt{\frac{(0.12-0.14)^2+(0.14-0.14)^2+(0.16-0.14)^2}{3-1}}$$

$$= \sqrt{\frac{0.0004+0+0.0004}{2}} = \sqrt{0.0004} = 0.02$$

Calculation of the expected return for each of the two portfolios:

| Year | Portfolio BI | Portfolio BG |
|---|---|---|
| 2004 | (0.12X0.50)+(0.16X0.50)=0.14 | (0.12X0.50)+(0.12X0.50)=0.12 |
| 2005 | (0.14X0.50)+(0.14X0.50)=0.14 | (0.14X0.50)+(0.14X0.50)=0.14 |
| 2006 | (0.16X0.50)+(0.12X0.50)=0.14 | (0.16X0.50)+(0.16X0.50)=0.16 |

Now, $\overline{X}_{BI} = \frac{14\%+14\%+14\%}{3} = 14\%$

$\overline{X}_{BG} = \frac{12\%+14\%+16\%}{3} = 14\%$

b. Calculation of the standard deviation for each portfolios:

$$\sigma_{BI} = \sqrt{\frac{\sum(X-X_{AB})^2}{n-1}} = \sqrt{\frac{(0.14-0.14)^2+(0.14-0.14)^2+(0.14-0.14)^2}{3-1}} = \sqrt{\frac{0}{2}} = 0$$

$$\sigma_{BG} = \sqrt{\frac{(0.12-0.14)^2+(0.14-0.14)^2+(0.16-0.14)^2}{3-1}} = \sqrt{\frac{0.0004+0+0.0004}{2}} =$$

$\sqrt{0.0004} = 0.02$

b. Portfolio AB is preferable because it presents the same return with less risk than Portfolio AC.