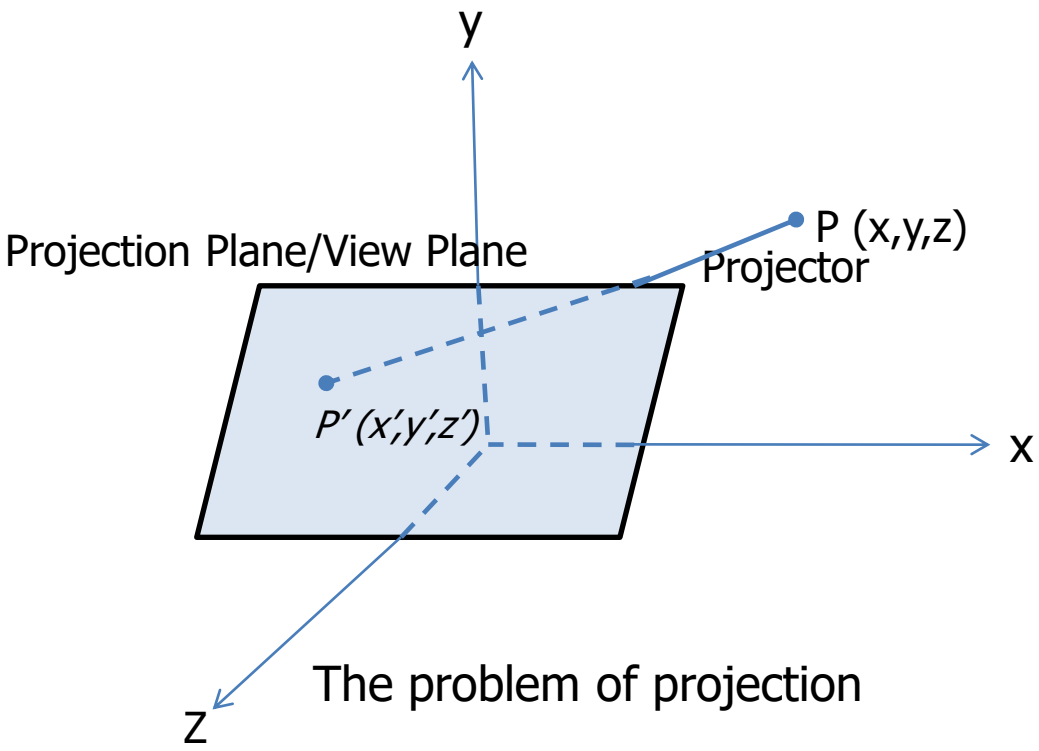
Computer Graphics

Lecture-09
Mathematics of Projection

Md Imtiaz Ahmed
Lecturer,
DIIT

Projections

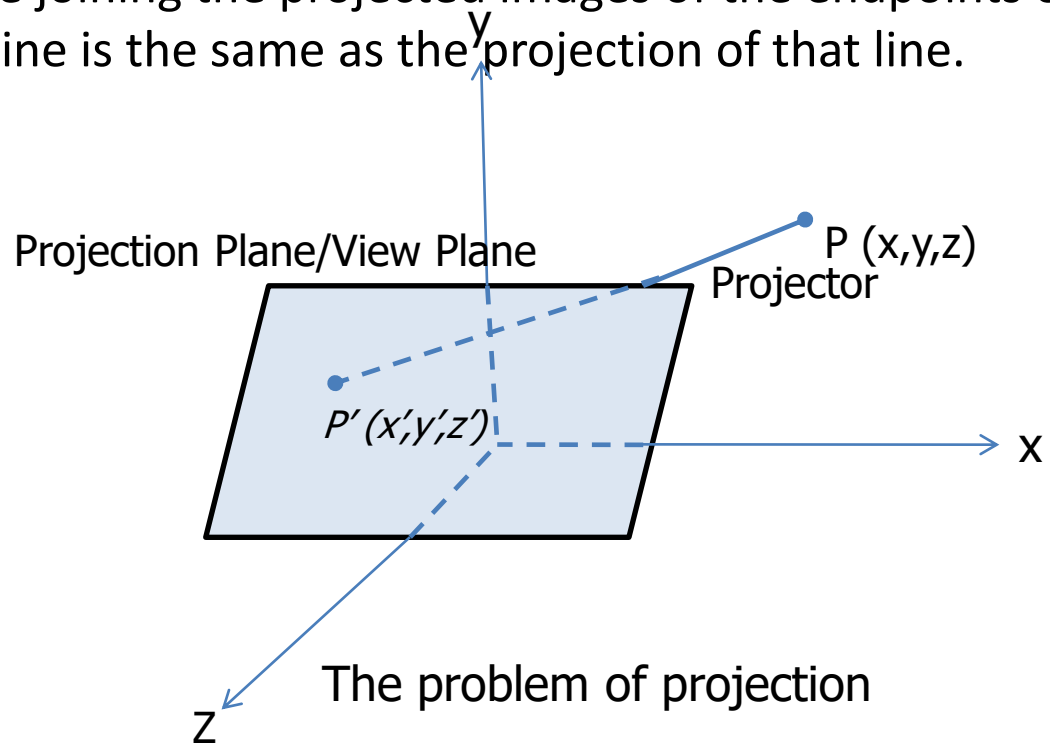
- Projection can be defined as a mapping of point P(x,y,z) on to its image P'(x',y',z') in the projection plane or view plane , which constitutes the display surface.
- The mapping is determined by a projection line called the projector that passes through P and intersects the view plane.
- The intersection point is P'



The problem of projection

Projections

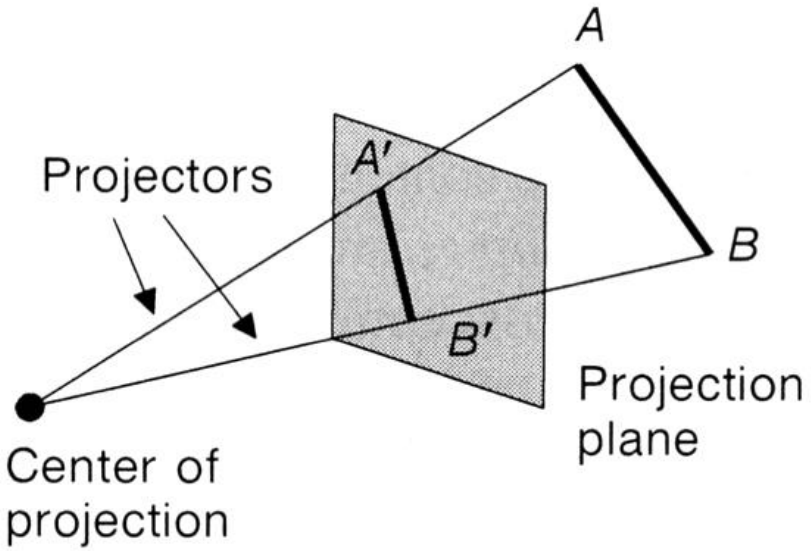
- The result of projecting an image is dependent on the spatial relationship among the projectors that project the points on the object , and the spatial relationship between the projectors and the view plane
- An important observation is that projection preserves the lines. That is the line joining the projected images of the endpoints of the original line is the same as the projection of that line.



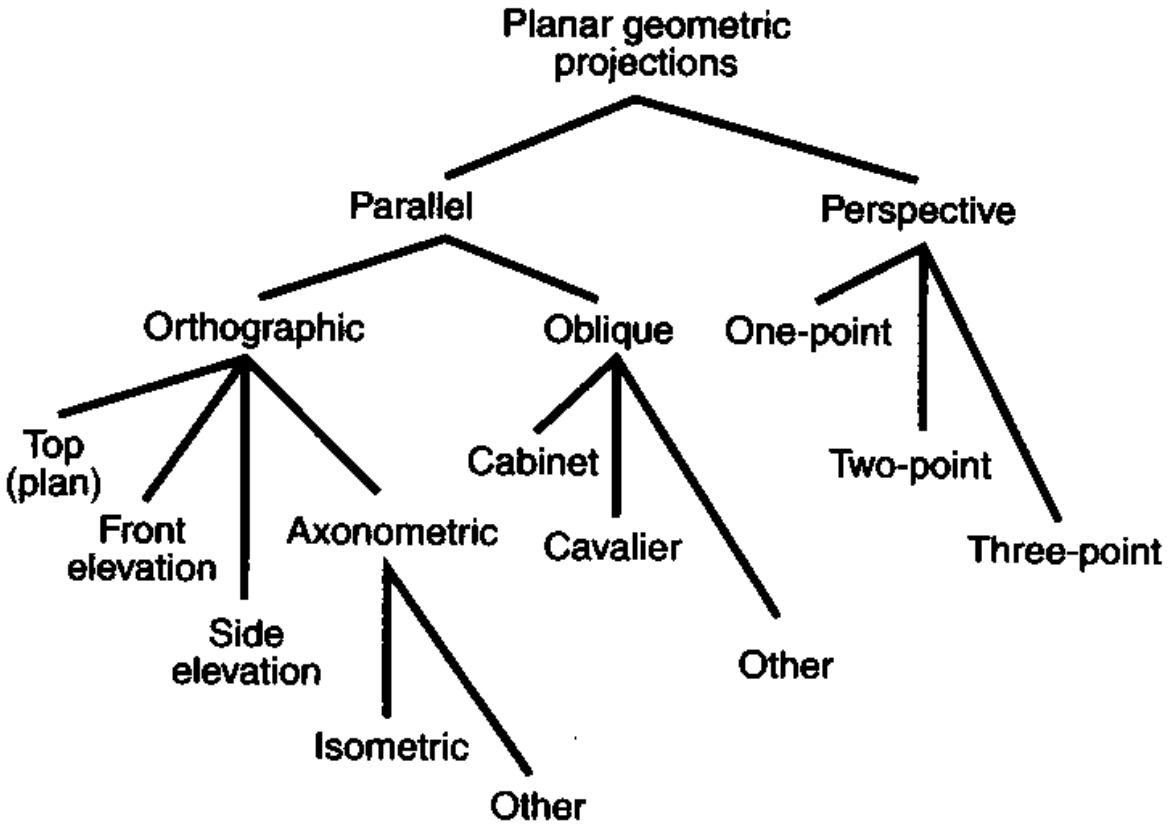
The problem of projection

Projections

- Projections: key terms…
  - *Projection* from 3D to 2D is defined by straight *projection rays* (*projectors*) emanating from the '*center of projection*', passing through each point of the object, and intersecting the '*projection plane or view plane*' to form a projection.
- Display device (a screen) is 2D…
  - How do we map 3D objects to 2D space?



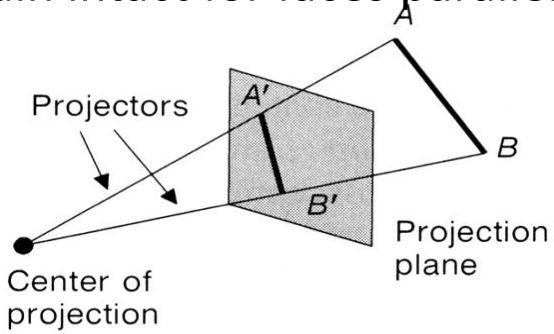
Overview of projections



Types of Projection

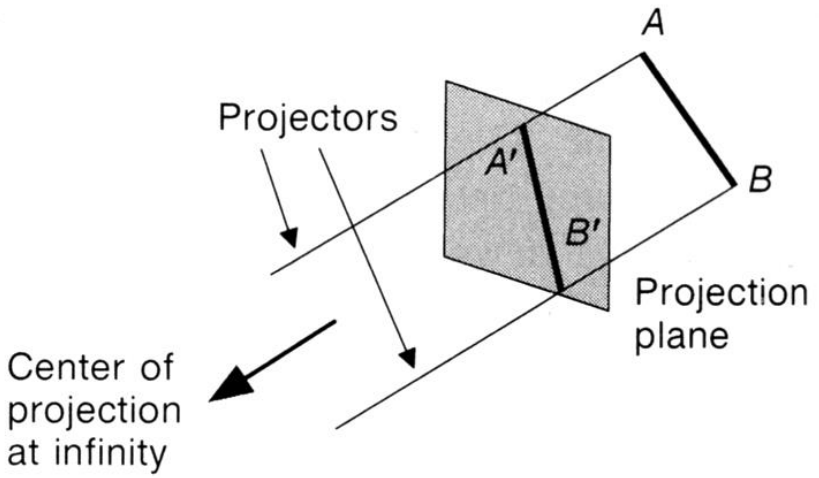
- Key factor is the *center of projection*.
  - if distance to center of projection is finite : perspective
  - if infinite : parallel
- Perspective projection
  - Projection lines are crossing the view plane and converge in a *projection reference point* (PRP)
  - An image point is determined by a projector that goes from an object point to the center of projection.
  - visual effect is similar to human visual system…
  - has 'perspective foreshortening'
    size of object varies inversely with distance from the center of projection.

—angles only remain intact for faces parallel to projection plane.



Types of Projection

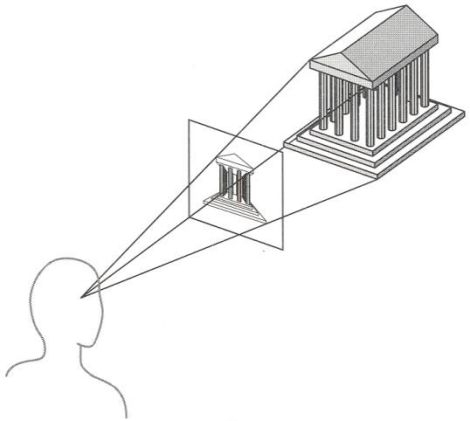
- Parallel projection
  - All projection lines are crossing the view plane in parallel; preserve relative proportions
  - Less realistic view because of no foreshortening
  - However, parallel lines remain parallel.
  - Angles only remain intact for faces parallel to projection plane.

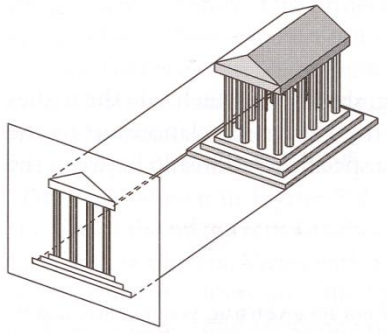


Perspective vs. Parallel

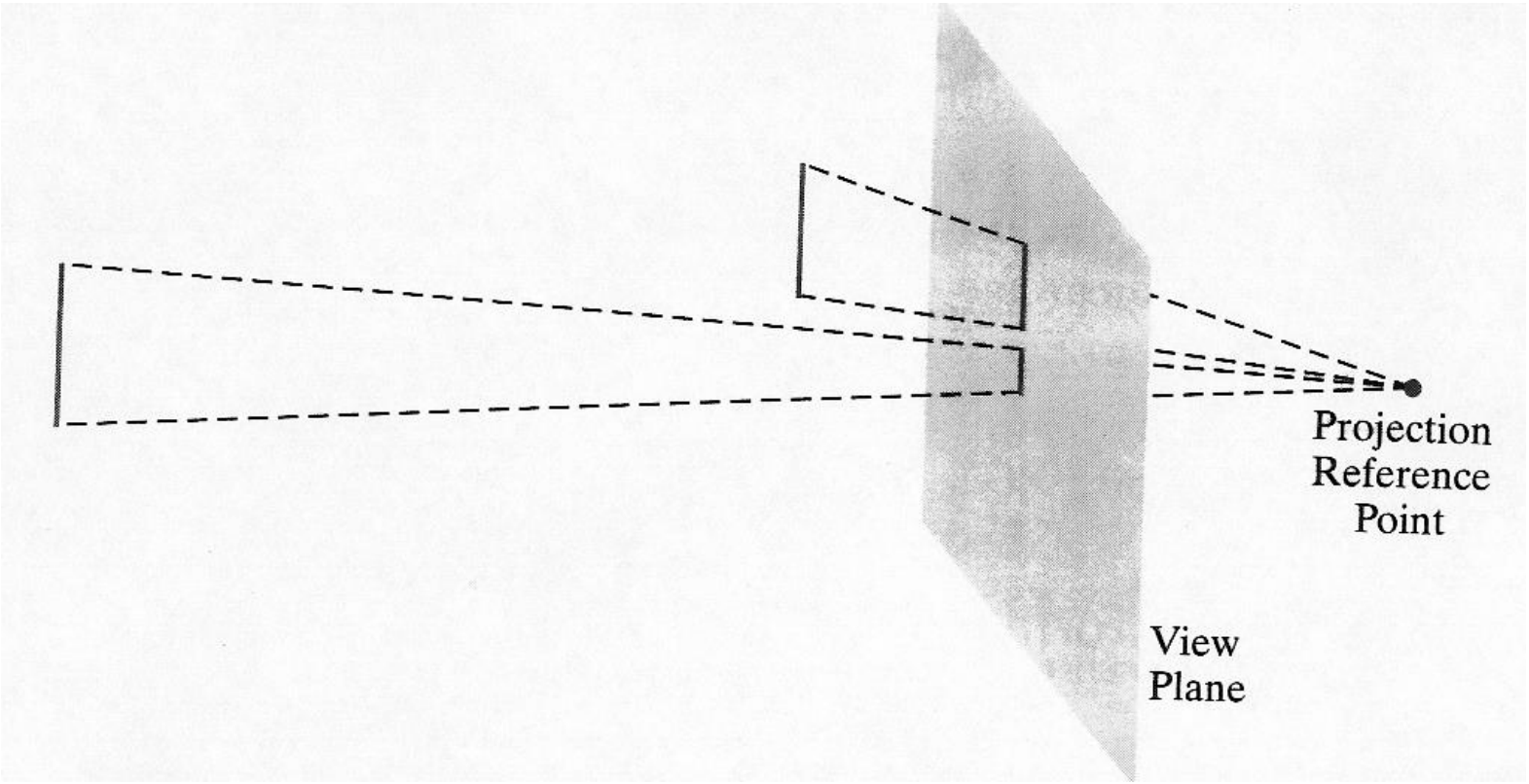
- Perspective projection
  - \+ Size varies inversely with distance - looks realistic
  - – Distance and angles are not (in general) preserved
  - – Parallel lines do not (in general) remain parallel

- Parallel projection
  - \+ Good for exact measurements
  - \+ Parallel lines remain parallel
  - – Angles are not (in general) preserved
  - – Less realistic looking





Perspective projection



Perspective Projections

- Any parallel lines *not* parallel to the projection plane, converge at a vanishing point.
  - There are an infinite number of these, 1 for each of the infinite amount of directions line can be oriented.

- If a set of lines are parallel to one of the three principle axes, the vanishing point is called an *axis vanishing point*.
  - There are at most 3 such points, corresponding to the number of axes cut by the projection plane.

Perspective Projections

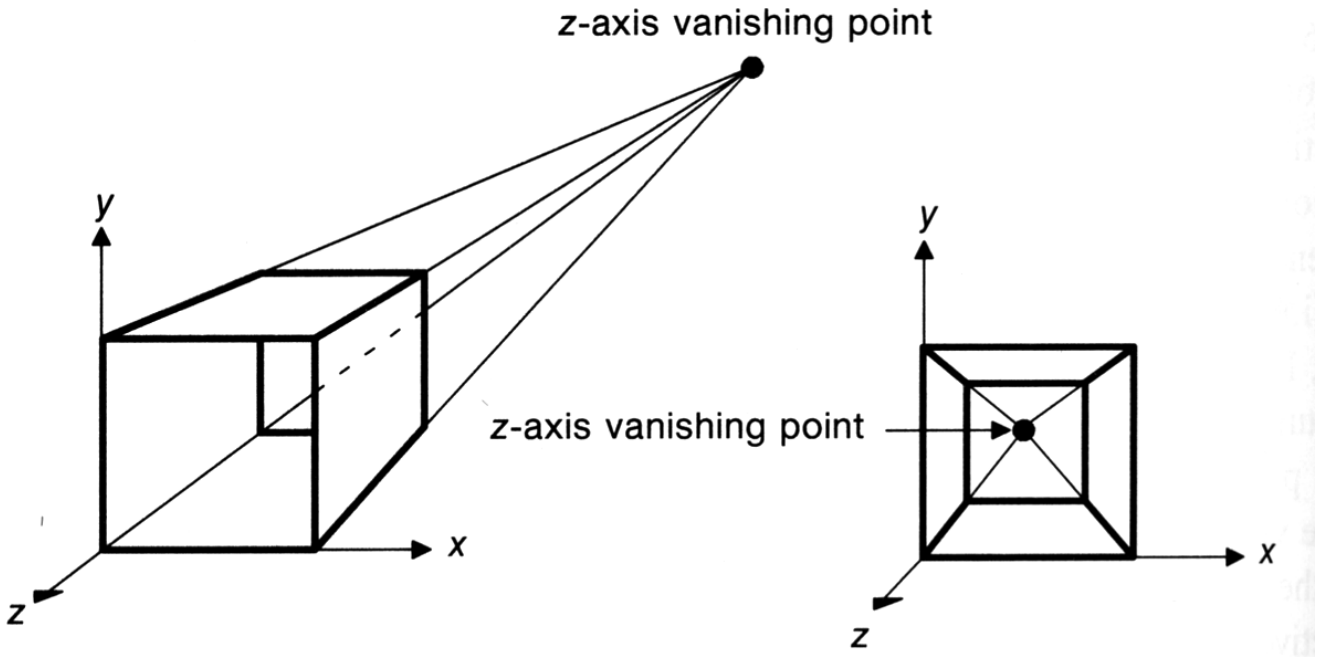
- Example:
  - if z projection plane cuts the z axis: normal to it, so only z has a principle vanishing point, as x and y are parallel and have none.

- Can categorise perspective projections by the number of principle vanishing points, and the number of axes the projection plane cuts.

Perspective Projections

- 2 different examples of a one-point perspective projection of a cube.

  (note: x and y parallel lines do not converge)

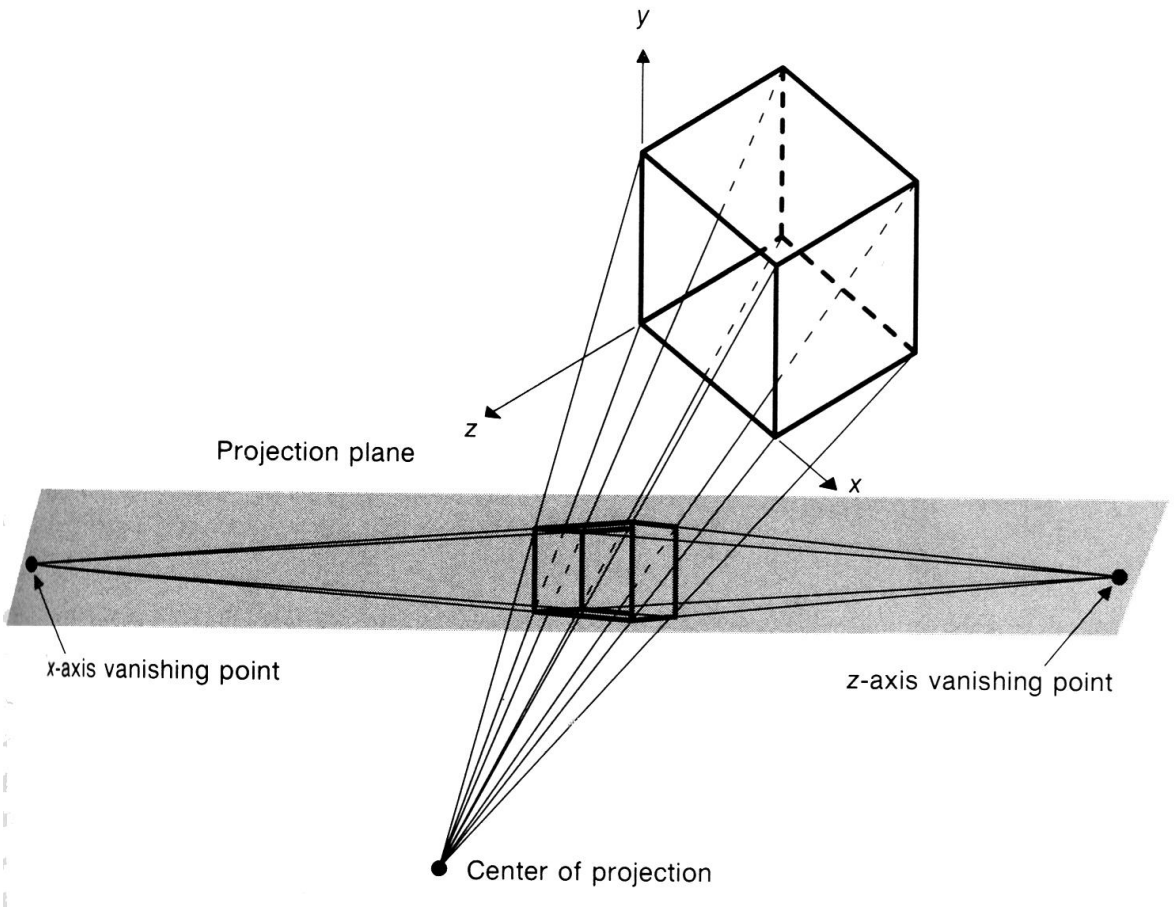


Perspective Projections

- Two-point perspective projection:
  - This is often used in architectural, engineering and industrial design drawings.
  - Three-point is used less frequently as it adds little extra realism to that offered by two-point perspective projection.

Perspective Projections

- Two-point perspective projection:



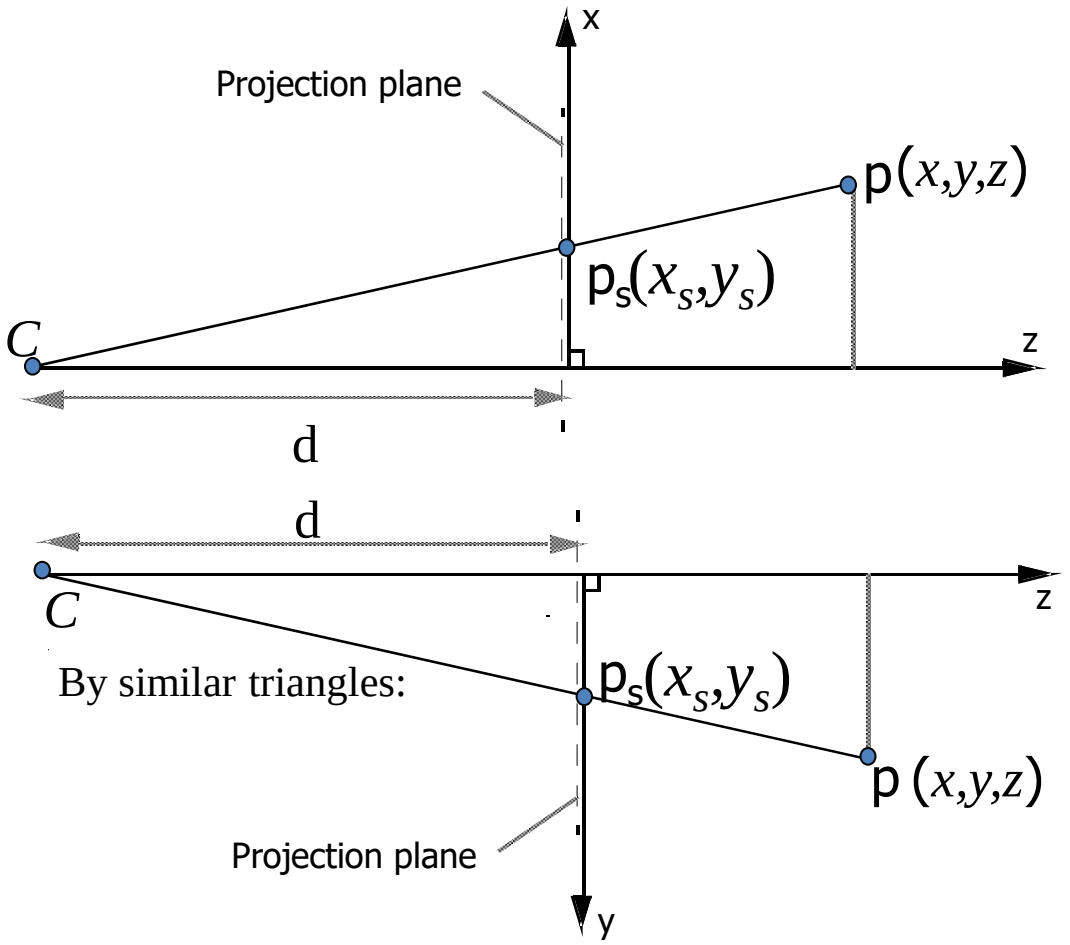
Perspective Projections

$$\frac{x}{z+d} = \frac{x_s}{d}$$

$$x_s = \frac{x}{1+\frac{z}{d}} = \frac{d.x}{z+d}$$

$$y_s = \frac{y}{1+\frac{z}{d}} = \frac{d.y}{z+d}$$

$$z' = 0$$



Perspective Projections

$$\begin{pmatrix} x_s \\ y_s \\ 0 \\ 1+\frac{z}{d} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_s \\ y_s \\ 0 \\ 1+\frac{z}{d} \end{pmatrix} = \begin{pmatrix} \frac{x_s}{1+\frac{z}{d}} \\ \frac{y_s}{1+\frac{z}{d}} \\ 0 \\ 1 \end{pmatrix}$$

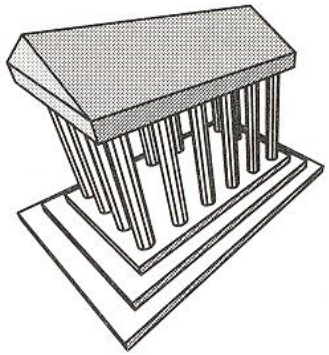
The general form of a Perspective Transformation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} d.x \\ d.y \\ 0 \\ z+d \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

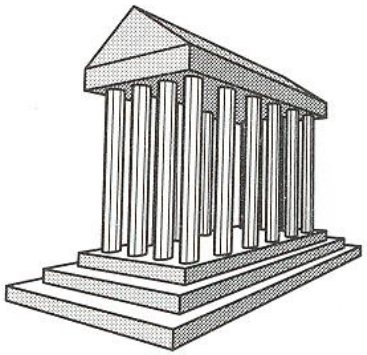
Road in perspective

Perspective Projection

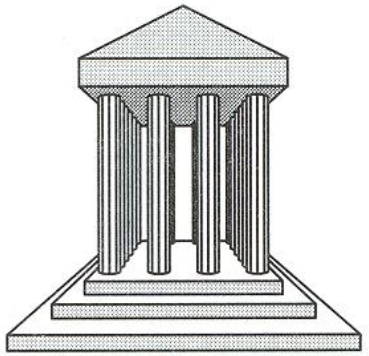
- How many vanishing points?



3-Point Perspective



2-Point Perspective

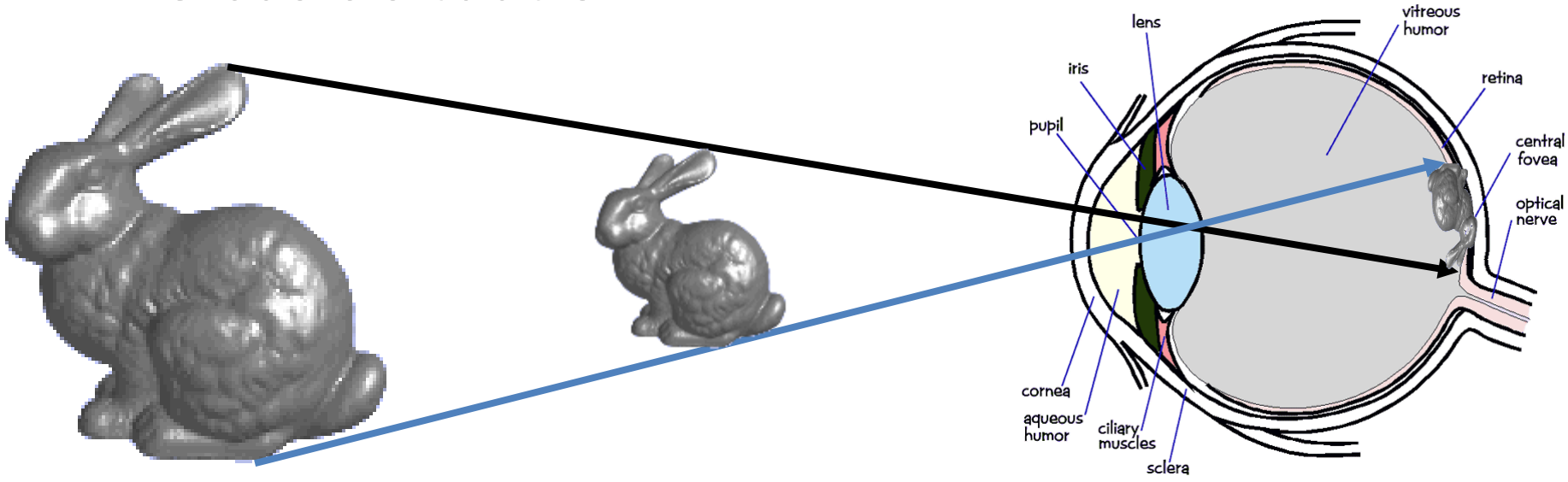


1-Point Perspective

Angel Figure 5.10

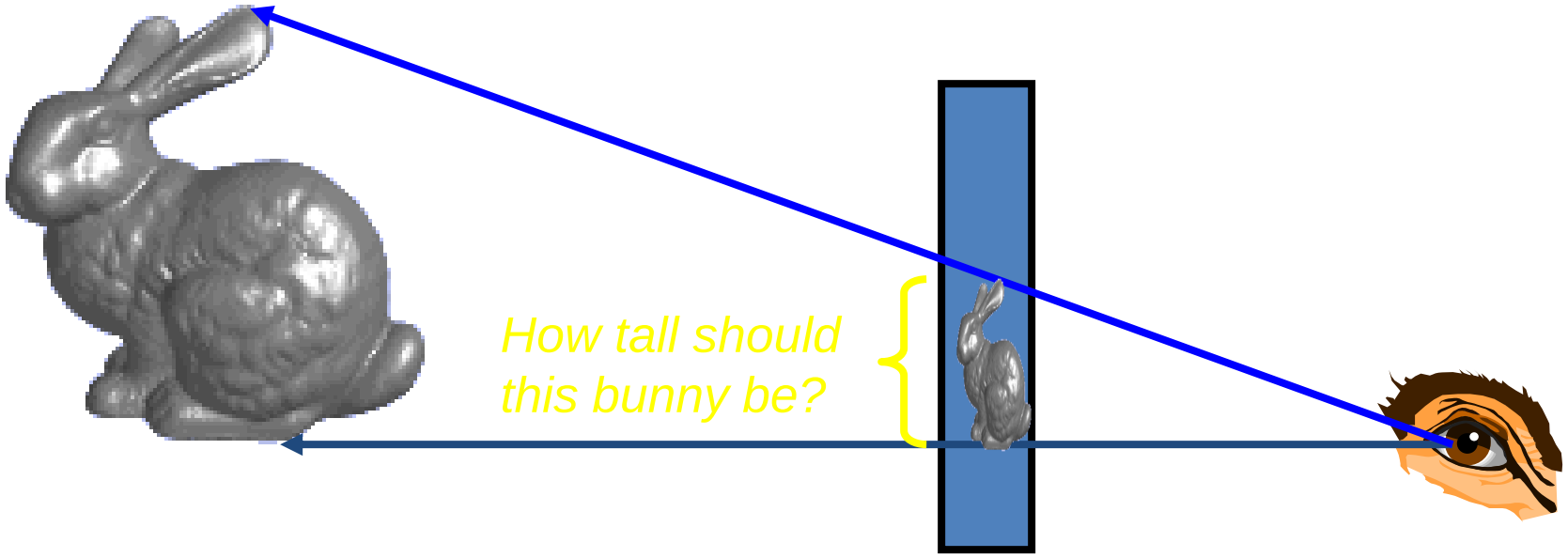
Perspective Projection

- In the real world, objects exhibit
  : distant objects appear smaller
- The basic situation:



Perspective Projection

- When we do 3-D graphics, we think of the screen as a 2-D window onto the 3-D world:



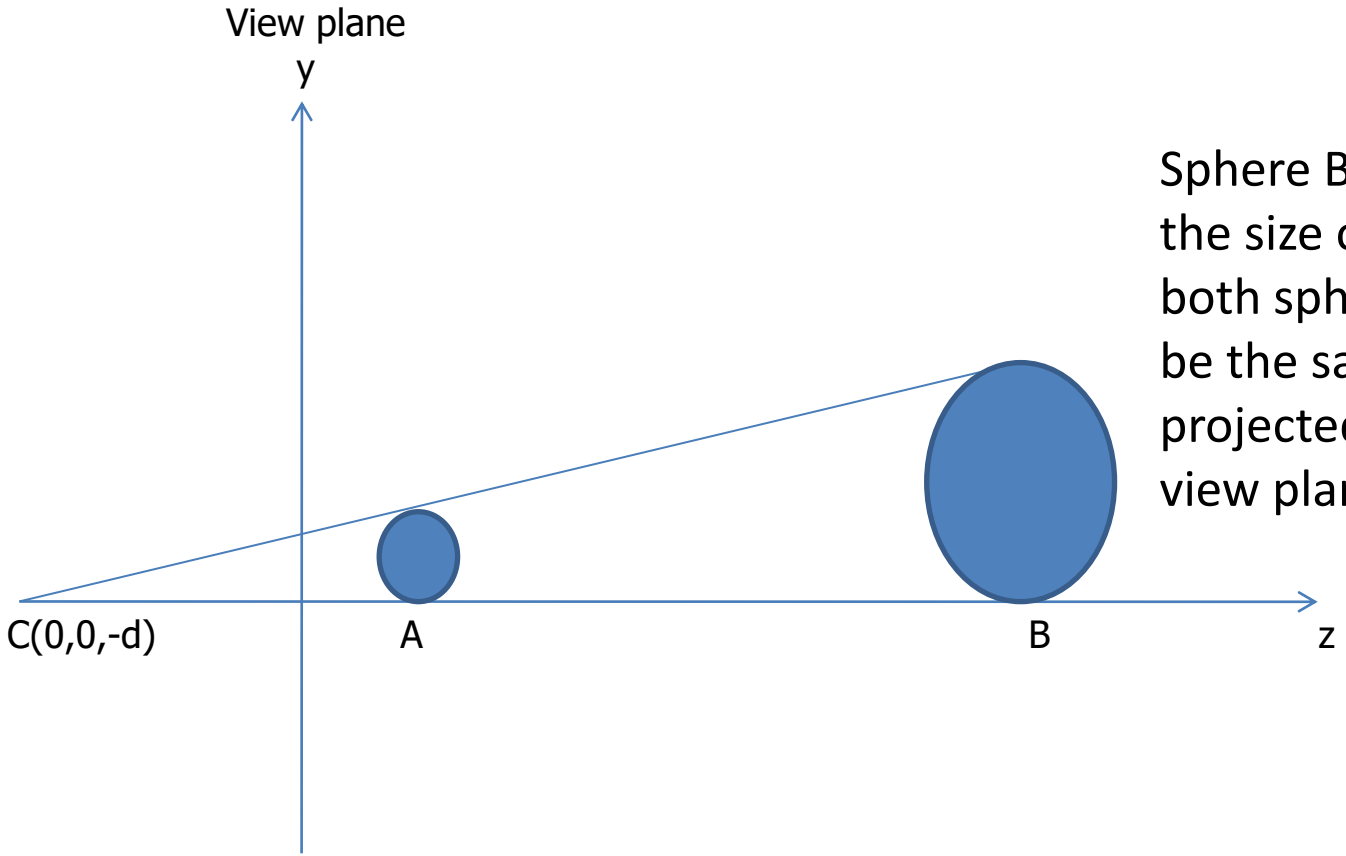
A 3D Scene

- Notice the presence of the camera, the projection plane, and the world coordinate axes

- Viewing transformations define how to acquire the image on the projection plane

Perspective anomalies

- Perspective foreshortening
  - The further an objective is from the center of projection, the smaller it appears (its projected size become smaller)



Sphere B is 2.5 times the size of sphere A, yet both spheres appear to be the same size when projected on to the view plane.

Perspective anomalies

- Vanishing points:
  - Projections of lines that are not parallel to the view plane(lines that are not perpendicular to the view plane normal) appear to meet some point on the view plane
  - A common manifestation of this anomaly is the illusion that railroad tracks meet at a point on the horizon.

Perspective anomalies

- View confusion:
  - Objects behind the center of projection are projected upside down and backward on to the view plane
- Topological distortion:
  - Consider the plane that passes through the center of projection and is parallel to the view plane.
    - The point of this plane are projected to infinity by the perspective transformation.
    - In particular a finite line segment joining a point which lies in front of the viewer to a point in back of the viewer is actually projected to a broken line of infinite extent.