Computer Graphics

Lecture-10
Mathematics of Projection

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Parallel Projection

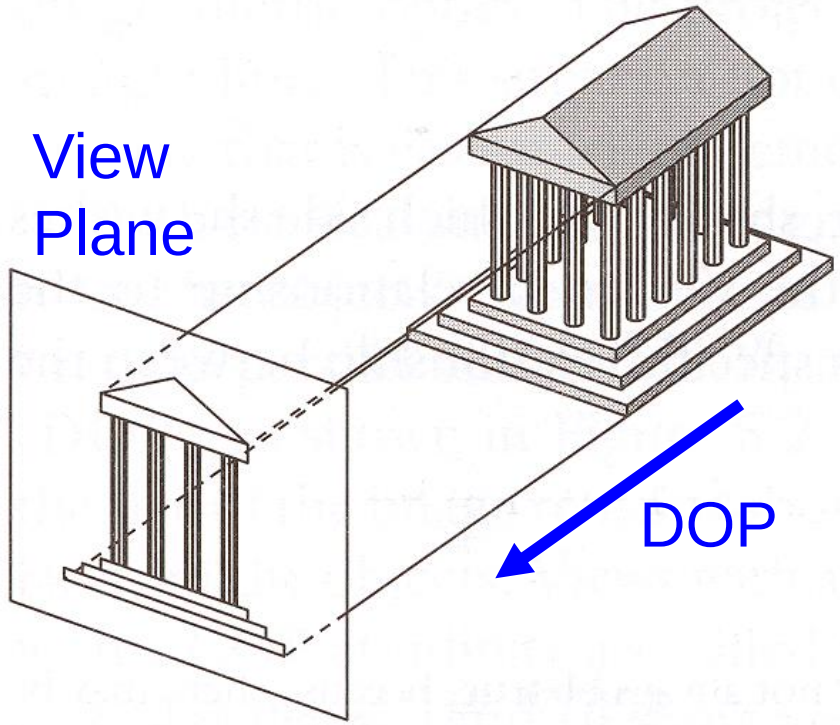
- **Parallel projection** methods are used by drafters and engineers to create working drawings of an object which preserves its scale and shape.
- The complete representation of these details often requires two or more views(projections) of the object onto different view planes.

Parallel Projection

- Image points are found as the intersection of the view plane with a projector drawn from the object point and having a fixed direction.
- The direction of projection is the prescribed direction for all projectors.

Parallel Projection

- Center of projection is at infinity
  - Direction of projection (DOP) same for all points



Angel Figure 5.4

Parallel Projections

- **Orthographic** :
  - Orthographic projections are characterized by the fact that the direction of projection is perpendicular to the view plane.

- **Oblique** : Non-orthographic parallel projections are called oblique parallel projections.

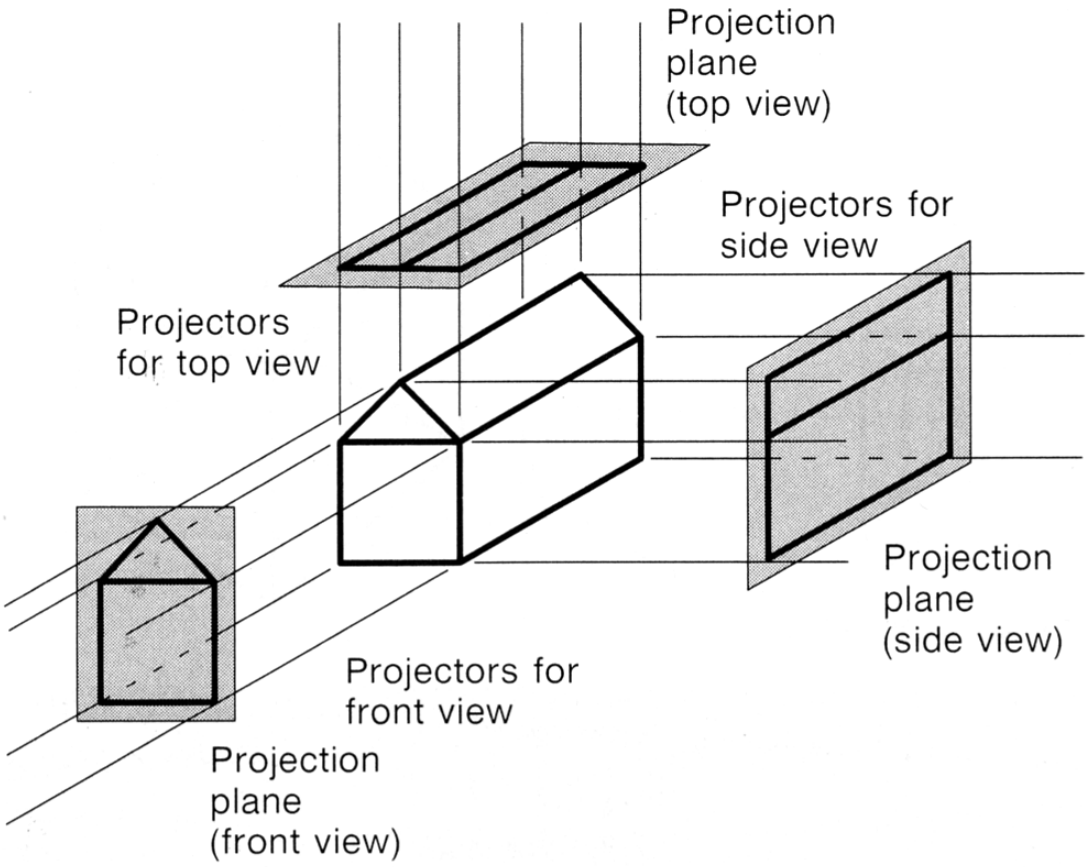
Orthographic Projections

- **Orthographic (or orthogonal) projections**:
  - front elevation, top-elevation and side-elevation.
  - all have projection plane perpendicular to a principle axes.

- Useful because angle and distance measurements can be made...

Orthographic Projections

- Orthogonal projections:



Orthographic Projections

- When the direction of projection is parallel to any of the principal axes, this produces the front, top, and side views of mechanical drawings (also referred to as **multiview** drawings).
- **Axonometric** projections are orthographic projections in which the direction of projection is not parallel to any of the three principal axes.

Axonometric projections

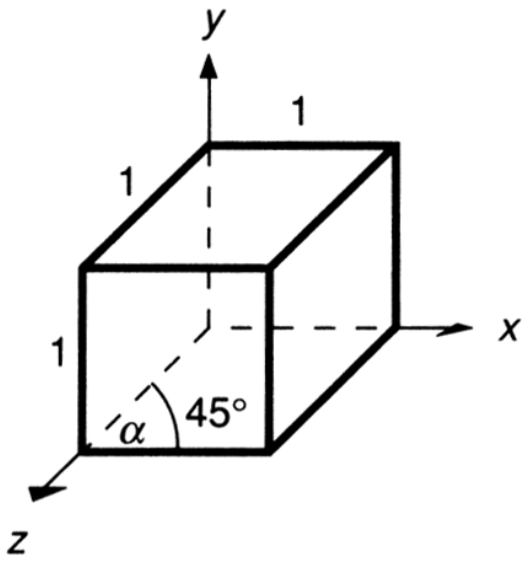
- **Isometric**: The direction of projection makes equal angles with all of the three principal axes.
- **Dimetric**: The direction of projection makes equal angles with exactly two of the principal axes.
- **Trimetric**: The direction of projection makes unequal angles with the three principal axes.

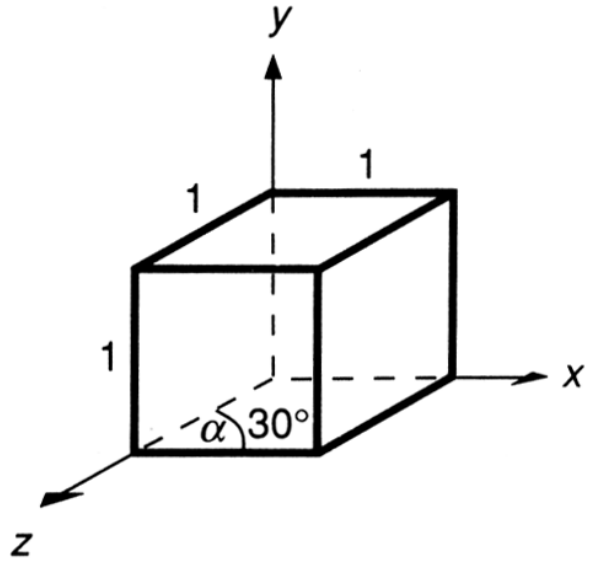
Oblique Projections

- **Oblique parallel projections**
  - Objects can be visualized better than with orthographic projections
  - Can measure distances, but not angles
  - Can only measure angles for faces of objects parallel to the plane
- **Common oblique parallel projections:**
  - *Cavalier* and *Cabinet*

Parallel Projections

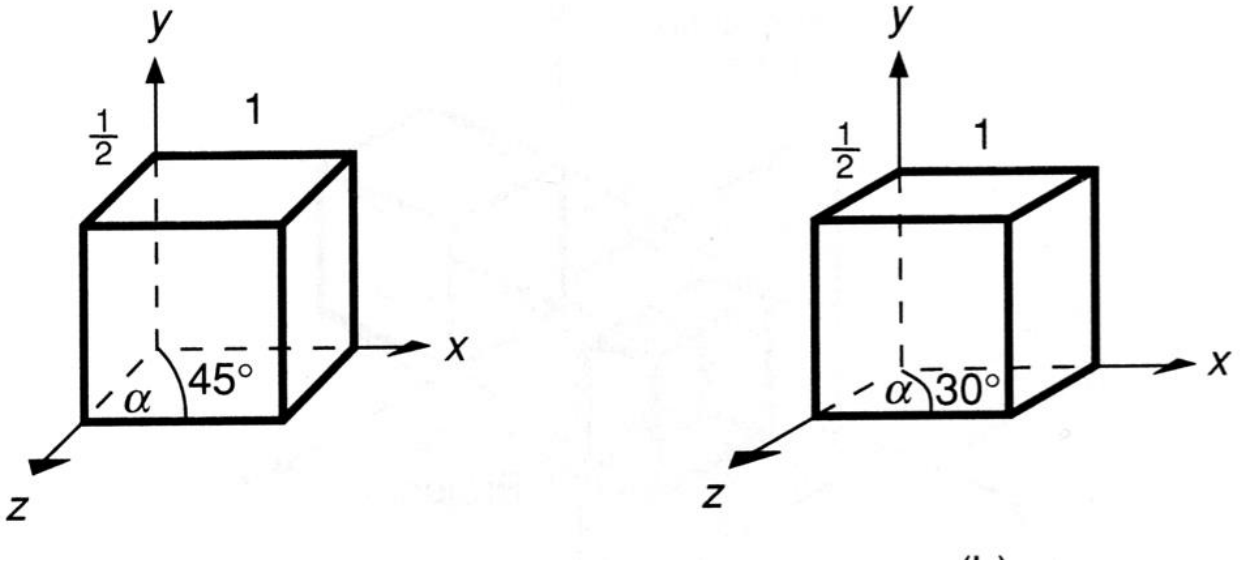
- **Cavalier**:
  - The direction of the projection makes a 45 degree angle with the projection plane.





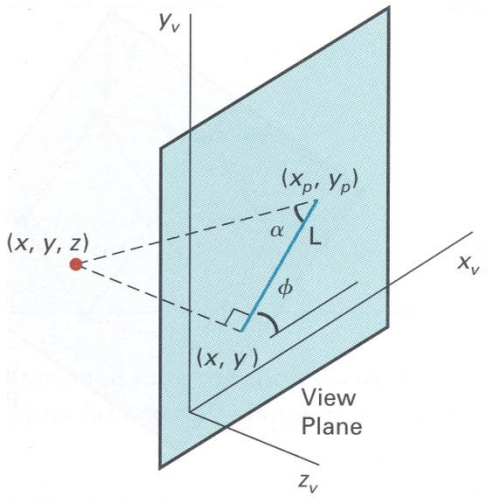
Parallel Projections

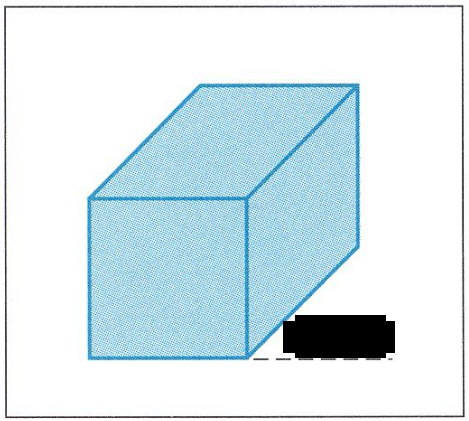
- **Cabinet**:
  - The direction of the projection makes a 63.4 degree angle with the projection plane. This results in foreshortening of the z axis, and provides a more “realistic” view.



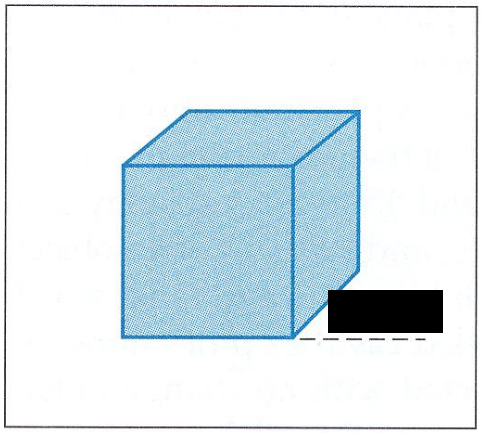
Oblique Projections

- DOP not perpendicular to view plane





Cavalier
(DOP $\alpha = 45^o$)
$\tan(\alpha) = 1$



Cabinet
(DOP $\alpha = 63.4^o$)
$\tan(\alpha) = 2$

Oblique Parallel Projections

- At (0,0,1)

  $x_s = \lambda \cos \alpha$

  $y_s = \lambda \sin \alpha$

  $z_s = 0$

- Generally
  - multiply by z and allow for (non-zero) x and y

$$x_s = x + z.\lambda.\cos \alpha$$

$$y_s = y + z.\lambda.\sin \alpha$$

Oblique Parallel Projections

$$\begin{pmatrix} x_s \\ y_s \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \lambda\cos\alpha & 0 \\ 0 & 1 & \lambda\sin\alpha & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$